# **Status of the Bonn-Gatchina partial wave analysis**

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Abstract. The Bonn-Gatchina partial wave formalism is extended to include a decomposition of t- and u-exchange amplitudes into individual partial waves. The multipole transition amplitudes for  $\gamma p \rightarrow p\pi^0$  and  $\gamma p \rightarrow n\pi^+$  are given and compared to results from other analyses.

**Keywords:** Baryon resonances, partial wave analysis, multipole amplitudes, photoproduction **PACS:** 11.80.-m; 13.60.-r; 13.75.-n; 14.20.-c

## **INTRODUCTION**

Most information about baryons comes from pion- and photon-induced production of single mesons. However the experience from meson spectroscopy shows that excited states decay dominantly into multi-body channels and are not observed reliably in the elastic cross section. Thus reactions with three or more final states provide rich information about the properties of hadronic resonances.

The task to extract pole positions and residues from multi-body final states is however not a simple one. Main problems can be traced to the large interference effects between different isobars and to contributions from singularities related to multi-body interactions. In our method, singularities in the reaction can be classified, resonances which are closest to the physical region can be taken into account accurately. Other contributions can be parameterized in an efficient way.

One of the key points in this approach is the operator decomposition method which provides a tool for a universal construction of partial wave amplitudes for reactions with two– and many–body final states.

### DATA USED IN THE FITS

A large number of reactions [1] is used in the coupled-channel fits presented here. The data cover elastic  $\pi N$  scattering as well as inelastic reactions, they cover differential cross sections and single and double polarization variables in photoproduction reaction. Reactions with multi-body final states are included exploiting an event-based likelihood method.

#### PARTIAL WAVE AMPLITUDES

A general expression for the decomposition of the two-particle scattering amplitude A(s,t) into partial wave amplitudes  $A_n^{\beta\beta'}(s)$  which describe production, propagation and decay of a two-particle systems with fixed total angular momentum *J*, parity and (if conserved) *C*-parity can be written as:

$$A(s,t) = \sum_{\beta\beta' n} A_n^{\beta\beta'}(s) Q_{\mu_1...\mu_n}^{(\beta)\dagger}(k) F_{\nu_1...\nu_n}^{\mu_1...\mu_n} Q_{\nu_1...\nu_n}^{(\beta')}(q)$$
(1)

where  $k_i$  are initial and  $q_i$  are final particle momenta,  $s = (k_1 + k_2) = (q_1 + q_2) = P^2$ ,  $t = (k_1 - q_1)^2 = (k_2 - q_2)^2$ ,  $k = (k_1 - k_2)/2$ ,  $q = (q_1 - q_2)/2$  and n = J for a boson system and n = J - 1/2 for a fermion one. The vertices  $Q_{v_1...v_n}^{(\beta)\dagger}$  and  $Q_{\mu_1...\mu_n}^{(\beta)\dagger}$  († \* stands for hermitian conjugation) describe the transition of the system into the initial- and final-state particles, and depend on the total and relative momenta. The indices  $\beta$  and  $\beta'$  list quantum numbers of the production and decay amplitudes, e.g. isospin, spin and orbital angular momenta. The tensor  $F_{v_1...v_n}^{\mu_1...\mu_n}$  depends only on

the total momentum P and describes the tensor structure of the partial wave. It is often called projection operator. The formalism for construction of vertices for meson-baryon partial waves and projection operators is given in [2, 3].

In the case of resonance production, the total amplitude A(s,t) can be expanded into a sum of partial wave amplitudes multiplied by vertices, see eq. (1). Here the partial wave amplitudes  $A_n^{\beta\beta'}(s)$  provide the energy dependence of the resonance which can be parameterized, for example, as N/D amplitude, as K-matrix or, in the simplest case, as a Breit-Wigner amplitude [4]. For non-resonant contributions, like t and u channel exchanges, the situation is different. In many partial wave analyses (including the present one) these contributions are simply added to the resonant part of the total amplitude and the sum is used to fit the experimental data. However, one needs to know the contribution of t and u-exchanges in every partial wave if the final partial wave amplitudes are to be compared with results from other analyses. This decomposition is also required when rescattering between non-resonant and resonant parts of the amplitude should be taken into account. For the non-resonant contributions used in the energy dependent fits one has therefore to solve an inverse task: to extract partial wave amplitudes from the total amplitude.

This task can be solved by using the orthogonality condition for partial wave operators. Multiplying the total amplitude from eq. (1) with initial and final projection operators and vertices and integrating over solid angle of the initial and final momenta we obtain

$$F_{\mu_{1}...\mu_{n}}^{\tau_{1}...\tau_{n}}\int \frac{d\Omega_{k}}{4\pi} \frac{d\Omega_{q}}{4\pi} Q_{\mu_{1}...\mu_{n}}^{(\alpha)}(k) A(s,t) Q_{\nu_{1}...\nu_{n}}^{(\alpha')}(q) F_{\eta_{1}...\eta_{n}}^{\nu_{1}...\nu_{n}} = (-1)^{n} F_{\eta_{1}...\eta_{n}}^{\tau_{1}...\tau_{n}} \sum_{\beta\beta'} A_{n}^{\beta\beta'}(s) W_{n}^{\alpha\beta}(k_{\perp}^{2}) W_{n}^{\beta'\alpha'}(q_{\perp}^{2}), \quad (2)$$

where  $k_{\perp}^2$  and  $q_{\perp}^2$  are squared relative momenta orthogonal to the total momentum of the system *P*. The factor  $W_n^{\alpha\beta}$  corresponds to the on-shell one-loop amplitude for transition between two vertices  $Q_{\mu_1...\mu_n}^{(\beta)}$ . It can be calculated as

$$W_{n}^{\alpha\beta}(k_{\perp}^{2}) = \frac{F_{\mu_{1}...\mu_{n}}^{\alpha_{1}...\alpha_{n}}}{\xi_{n}} \int \frac{d\Omega_{k}}{4\pi} Q_{\mu_{1}...\mu_{n}}^{(\alpha)}(k) Q_{\nu_{1}...\nu_{n}}^{(\beta)}(k) F_{\alpha_{1}...\alpha_{n}}^{\nu_{1}...\nu_{n}}$$
  
$$\xi_{n} = (-1)^{n} F_{\mu_{1}...\mu_{n}}^{\nu_{1}...\nu_{n}} g_{\mu_{1}\nu_{1}} \dots g_{\mu_{n}\nu_{n}}.$$
 (3)

For meson-nucleon and  $\gamma N$  vertices, the  $W_n^{\alpha\beta}$  were calculated in [3].

# **PHOTOPRODUCTION MULTIPOLES**

Let us discuss the partial wave amplitudes. It should be stressed that the amplitudes we give for  $\gamma p \rightarrow p \pi^0$  and  $\gamma p \rightarrow n\pi^+$  are constrained by a large number of other reactions. This is particularly important in the vicinity of thresholds. Of course, the elastic  $\pi N$  scattering amplitude and the pion photoproduction amplitude are influenced by opening new channels and the couplings to the new channels can be estimated from their effect on the scattering and photoproduction amplitudes. But this is rather indirect, and it is desirable to take the inelastic channels into account directly.

The multipoles for  $\pi^0$  photoproduction are shown in Fig. 1 in comparison to the SAID SP09K2700 [5] and MAID 2007 [6] solutions, those for  $\gamma p \to \pi^+ n$  in Fig. 2. The errors cover a large number of fits which differ mostly by the parameterization of the  $2\pi N$  channel at masses above 1.8 GeV. Most amplitudes derived within the SAID, MAID, or BnGa approach yield consistent results, at least qualitatively. The best agreement is found for the  $M_1^+$  amplitude which describes the spin flip amplitude for the photo-induced transition from the proton to the  $\Delta$  resonance and its excitations. The  $\Delta$  resonance is fully elastic, hence the agreement in the low-mass region is not unexpected. Even the small  $E_1^+$ multipoles are not inconsistent. Some multipoles which we discuss next show significant differences between the different approaches. The  $E_0^+$  multipole has a similar structure in all three approaches but shows significant differences in detail. In the BnGa solution, the electric dipole transition  $E_0^+$  exceeds the other solutions in the threshold regions, likely due to a larger role of the subthreshold  $\Lambda K^+$  amplitude. The differences are even larger for the  $M_1^-$  multipole; this may be not unexpected in view of the notorious difficulties with the  $1/2^+$  partial wave. Surprisingly, the multipoles for  $\gamma p \rightarrow n\pi^+$  are in much better consistency. The differences in the  $E_2^-$  and  $M_2^-$  can be assigned to additional  $\Delta_{3/2^-}(1940)$ and  $\Delta_{3/2^-}(2260)$  resonances introduced to fit data on  $\gamma p \rightarrow p \pi^0 \eta$  [7, 8]. Significantly different are the multipoles leading to  $5/2^-$  states. In our fits, the  $E_2^+$  and  $M_2^+$  multipoles include an additional resonance  $N_{5/2^-}(2060)$  [9].



**FIGURE 1.** The real (two left-hand columns) and imaginary (two right-hand columns) part of multipoles for the  $\pi^0$  photoproduction. The errors are systematic and cover a large number of fits (see the text). The dashed curves correspond to the SAID solution SP09K2700 [5] and the dotted curves to the MAID solution 2007 [6]

# Helicity amplitudes

In Table 1 I compare the results on  $A_{1/2}$  and  $A_{3/2}$  for  $N^*$  and  $\Delta^*$  with previous determinations of these quantities. These real helicity amplitudes are given in Table 1 and compared to values obtained by SAID and MAID, and to the values listed by the PDG [10]. First let us notice that our errors are much larger than those given by FA08. It seems that the FA08 systematic errors are underestimated: the impact of variations in the couplings to inelastic channels can hardly be tested using only reactions with  $N\pi$  in the final state. The errors we quote are not statistical errors; those are small. Our errors are derived from a large number of fits changing the number of resonances, switching on and off couplings to inelastic channels, using different start values for the fits.

For most resonances, reasonable consistency between the different analyses is found. So let us comment briefly on the differences. The PDG result for the  $A_{1/2}$  amplitude of  $(53\pm16) \text{ GeV}^{-1/2} \times 10^3$  for producing  $S_{11}(1650)$  was driven by the 1995 VPI result  $(69\pm5) \text{ GeV}^{-1/2} \times 10^3$  [11] and by the small value  $(22\pm7) \text{ GeV}^{-1/2} \times 10^3$  obtained in [12]. The most recent FA08 analysis gives  $(9.0\pm9.1) \text{ GeV}^{-1/2} \times 10^3$ , a value which is much smaller and which is not confirmed here; we find  $(60\pm20) \text{ GeV}^{-1/2} \times 10^3$ . Part of the discrepancy is certainly due to the  $S_{11}(1650)$  branching ratio to the  $\pi N$  channel; in FA09 this is fixed to be 100% while we find  $(50\pm25)\%$ . Of course, photoproduction defines only the product of the helicity and  $\pi N$  couplings.

Possibly related are the differences in the helicity amplitudes for  $P_{13}(1720)$ . Our value for  $A_{1/2}$  is compatible with the new FA08 analysis and in conflict with the value quoted by the PDG. Incompatible with all other determinations - even in the sign - is our value for the  $A_{3/2}$  helicity amplitude for  $P_{13}(1720)$  production. Clearly, more data are required to resolve this discrepancy; the results from double polarization experiments carried out at present in different laboratories will very likely be decisive.



**FIGURE 2.** The real (two left-hand columns) and imaginary (two right-hand columns) part of multipoles for the  $\gamma p \rightarrow \pi^+ n$  reaction. The errors are systematic and cover a large number of fits (see the text). The dashed curves correspond to the SAID solution SP09K2700 [5] and the dotted curves to the MAID solution 2007 [6].

Resonance	$A_{1/2} ({\rm GeV}^{-1/2} \times 10^3)$				$A_{3/2}$ (GeV <sup>-1/2</sup> ×10 <sup>3</sup> )			
	BnGa09	FA08	MAID07	PDG	BnGa09	FA08	MAID07	PDG
$S_{11}(1535)$	$90 \pm 15$	$100.9 \pm 3.0$	66	$90{\pm}30$				
$S_{11}(1650)$	$60{\pm}20$	9.0±9.1	33	53±16				
$P_{11}(1440)$	$-52 \pm 10$	$-56.4{\pm}1.7$	-61	$-65\pm4$				
$P_{11}(1710)$	$25\pm10$			$9\pm22$				
$P_{13}(1720)$	$130{\pm}50$	90.5±3.3	73	$18\pm30$	$100{\pm}50$	$-36.0{\pm}3.9$	-11	$-19{\pm}20$
$D_{13}(1520)$	$-32\pm6$	$-26{\pm}1.5$	-27	$-24\pm9$	$138{\pm}8$	$141.2 \pm 1.7$	161	$166\pm5$
$D_{15}(1675)$	$21\pm4$	$14.9 {\pm} 2.1$	15	$19\pm8$	$24\pm8$	$18.4{\pm}2.1$	22	$15\pm9$
$F_{15}(1680)$	$-12\pm 6$	$-17.6{\pm}1.5$	-25	$-15\pm6$	$136{\pm}12$	$134.2{\pm}1.6$	134	$133{\pm}12$
$S_{31}(1620)$	63±12	47.2±2.3	66	27±11				
$P_{33}(1232)$	-136±5	$-139.6{\pm}1.8$	-140	$-135{\pm}6$	$-267 \pm 8$	$-258.9{\pm}2.3$	-265	$-250\pm8$
$D_{33}(1700)$	$160{\pm}45$	$118.3 \pm 3.3$	226	$104{\pm}15$	$160{\pm}40$	$110.0{\pm}3.5$	210	85±22
$F_{35}(1905)$	$28 \pm 12$	$11.4{\pm}8.0$	18	26±11	$-42 \pm 15$	$-51.0{\pm}8.0$	-28	$-45{\pm}20$
or:	(48±12)				(0±3)			
$F_{37}(1950)$	-83±8	$-71.5{\pm}1.8$	-94	$-76{\pm}12$	$-92\pm8$	-96±8	-121	$-97{\pm}10$
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**TABLE 1.** Helicity amplitudes  $A_{1/2}$  and  $A_{3/2}$  for  $N^*$  and  $\Delta^*$  from this work, from SAID08 [5], from MAID07 [6], and estimates from Ref. [10].

#### SUMMARY

I have presented results from a partial wave analysis on a large variety of different reactions, from  $\pi N$  elastic scattering to photoproduction of multibody final states. The main emphasis of this paper was devoted to a determination of the electric and magnetic multipoles leading to the production of neutral or charged pions in photo-induced reactions off protons. The multipoles are mostly consistent with previous analyses but a few significant discrepancies call for clarifications. The analysis provides masses, widths, and helicity amplitudes for several known resonances. Masses and widths and the  $\pi N$  partial decay widths of all resonances agree very well with established values. Only the photocoupling of the  $P_{13}(1720)$  resonance differs remarkably from PDG and from the values found in a recent analysis of the CLAS collaboration.

The new data also require  $P_{11}(1710)$ . In [13], this resonance improved the description of the data slightly but we were not forced to introduce it. In the present fit, there are three resonances above the nucleon in the  $P_{11}$  wave: the Roper resonance  $P_{11}(1440)$ , the  $P_{11}(1710)$ , and the newly proposed  $P_{11}(1860)$ .

#### REFERENCES

- 1. A. V. Anisovich, E. Klempt, V. A. Nikonov, M. A. Matveev, A. V. Sarantsev and U. Thoma, arXiv:0911.5277 [hep-ph].
- 2. A.V. Anisovich et al., Eur. Phys. J. A 24, 111 (2005).
- 3. A.V. Anisovich and A.V. Sarantsev, Eur. Phys. J. A 30 (2006) 427.
- 4. A. V. Anisovich, V. V. Anisovich, M. A. Matveev, V. A. Nikonov, J. Nyiri and A. V. Sarantsev, "Mesons and baryons: Systematization and methods of analysis," *Hackensack, USA: World Scientific (2008) 580 pages.*
- 5. GWDAC.PHYS.GWU.EDU/
- 6. WWW.KPH.UNI-MAINZ.DE/MAID/
- 7. I. Horn et al., Phys. Rev. Lett. 101, 202002 (2008).
- 8. I. Horn et al., Eur. Phys. J. A 38, 173 (2008).
- 9. A.V. Anisovich et al., Eur. Phys. J. A 25 (2005) 427.
- 10. C. Amsler et al., Phys. Lett. B 667, 1 (2008).
- 11. R. A. Arndt, I. I. Strakovsky and R. L. Workman, Phys. Rev. C 53, 430 (1996).
- 12. M. Dugger et al., Phys. Rev. C 76, 025211 (2007).
- 13. A. V. Sarantsev et al., Phys. Lett. B 659, 94 (2008).