# Baryon resonances and strong QCD

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*Abstract: Light-baryon resonances (with u,d, and s quarks in the SU(3) classification) fall on Regge trajectories. When their squared masses are plotted against the intrinsic orbital angular momenta* L, $\Delta^*$*'s with even and odd parity can be described by the same Regge trajectory. For a given* L, *nucleon resonances with spin* S=*3/2 are approximately degenerate in mass with* $\Delta$ *resonances. To which total angular momentum* L *and* S *couple has no significant impact on the baryon mass. Nucleons with spin 1/2 are shifted in mass; the shift is - in units of squared masses - proportional to the component in the wave function which is antisymmetric in spin and flavor. Based on these observations, a new baryon mass formula is proposed which reproduces nearly all known baryon masses. It is shown that the masses are compatible with a quark-diquark picture while the richness of the experimentally known states require three particles to participate in the dynamics. This conflict is resolved by proposing that quarks polarize the QCD condensates and are surrounded by a polarization cloud shielding the color. A new interpretation of constituent quarks as colored quark clusters emerges; their interaction is responsible for the mass spectrum. Fast flavor exchange between the colored quark clusters exhausts the dynamical richness of the three-particle dynamics.*

*The colored-quark-cluster model provides a mechanism in which the linear confinement potential can be traced to the increase of the volume in which the condensates are polarized. The quark-spin magnetic moment induces currents in the polarized condensates which absorb the quark-spin angular momentum: the proton spin is not carried by quark spins. The model provides a new picture of hybrids and glueballs.*

# 1 Introduction

Baryon spectroscopy has played a decisive role in the development of the quark model and of flavor SU(3). The prediction of the $\Omega^-$ carrying total strangeness $S = -3$ [1] and its subsequent experimental discovery at the anticipated mass [2] was a triumph of SU(3). From the demand that the baryon wave function be antisymmetric with respect to the exchange of two quarks, the need of a further quark property was deduced [3] which later was called color [4] and found to play an eminent dynamical role. The linear dependence of the squared masses of baryons on their total angular momentum led to the Regge theory of complex angular momenta [5]. The unsuccessful attempts to 'ionize' protons and to observe free quarks [6] was the basis for the confinement hypothesis [7].

In this paper a phenomenological analysis of the spectrum of light baryons is presented. The three quarks with flavor up, down and strange may combine to the ground state octet baryons with total angular momentum J=1/2 or to the decuplet carrying angular momentum 3/2. These are the 'ground states' even though some of them have large hadronic widths: the $\Delta(1232)$, e.g., has a width of 120 MeV. Today, the Particle Data Group lists about 100 baryon resonances, 85 of them have an experimentally determined spin and parity, 50 baryon resonances are well established, having 3* or 4* in the PDG notation [8]. There is hence the hope, that a systematic investigation of the baryon masses reveals the internal interactions between quarks in the confinement region [9].

The study presented here shows that the orbital angular momentum is decisive for the excitation energies. The squared baryon masses depend linearly on the intrinsic orbital angular momentum; spin-orbit and spin-spin splittings due to color-magnetic forces are negligible. The well-known N-$\Delta$ splitting is traced to instanton-induced interactions. The observations can be condensed into a new baryon mass formula containing four parameters. The mass formula reproduces most known baryon masses with good accuracy.

The slope of the Regge trajectories of baryons is the same as the mesonic Regge slope. In the 70'ties, baryons could be divided into L-even baryons in SU(6) 56-plets, and L-odd baryons in 70-plets. These observations supported the assumption that quark-diquark interactions are responsible for the baryon masses [10]. A SU(6) classification of the now-existing baryon resonances will show that the mass spectrum is much richer and reflects the full freedom of three-particle dynamics.

Now there is a clear-cut contradiction: the richness of the mass spectrum evidences that three particles take place in the interaction. This is of course what we would have anticipated. On the other hand, the masses are all well described in a quark-diquark picture. These experimental findings suggest a new definition of 'constituent' quarks as colored clusters in which current-quarks polarize the $q\bar{q}$ and gluon condensates of the QCD vacuum. The color of the current quark is shielded by the polarization cloud; the polarization cloud absorbs gluons before they propagate to another current quark. The condensates themselves become colored, as suggested by Wetterich and collaborators [11]. The colored quark-clusters define the interaction. Each quark experiences the forces exerted by a colored diquark. The forces are hence equivalent to quark-antiquark forces. Quark flavor is exchanged with a high frequency; to first order, all SU(6) states of given intrinsic spin and orbital angular momenta and with the same quark content are degenerate in mass. Mass splitting occurs due to the spin- and flavor-dependence of instanton-induced interactions.

The new definition of a constituent quark has significant impact on the comprehension of strong interactions. It suggests an interpretation of the confinement mechanism similar to that proposed by Nambu [12]. It provides a natural interpretation of the proton *spin crisis*: as often discussed, only a small fraction of the proton spin can be ascribed to quarks (including the sea quarks) [13], the largest contribution to the proton spin must be assigned to orbital angular momenta of quarks and gluons, or to the gluon spin. In the new picture proposed here, the quark spin induces magnetic currents in the polarized condensates; the contribution of constituent quarks to the nucleon spin is small. For gluonic excitations, hybrids and glueballs, a new interpretation is proposed.

The outline of this paper is as follows: after a short introduction into the ideas behind present quark-models of baryons, we give an outline of SU(6) and of the symmetries of baryonic wave functions. In section 4, phenomenological aspects of the light-baryon spectra are presented. Regge trajectories are exploited to demonstrate the most important forces responsible for the baryon mass pattern. Based on these results a new baryon mass formula is proposed (section 5) and compared to data. The comparison of experimental masses and predictions of the model requires to assign baryons to SU(6) multiplets; this assignment is discussed in section 6.

The new mass formula requires a different concept of the constituent quark and a new interpretation of the QCD forces. This aspect is discussed in section 7. The paper ends with a short summary.

# 2 Baryon models

It has been stressed very often that the strong-interaction coupling-constant $\alpha_s$ increases dramatically and diverges when the momentum transfer q approaches the QCD scale parameter $\Lambda_{\rm QCD}$. This is the regime of strong QCD where perturbative methods fail. Bound states of quarks, mesons and baryons, involve very small momentum transfers; models need to be developed to appreciate the meaning of strong interactions in this low-energy domain.

There are a few distinct classes of baryon models based on different quark-quark and quark-antiquark interactions. Most models start from the assumption that QCD generates a confinement potential which grows linearly with the distances between the quarks. The color-degrees-of-freedom guarantee the antisymmetry of the baryon wave function. The equation of motion is solved after the color-degrees-of-freedom have been integrated out: color plays no dynamical role in the interaction. The confinement potential corresponds to the mean potential energy experienced by a quark at a given position, with a fast color exchange between the three quarks. Confinement does not exhaust the full QCD interaction: there are residual interactions which can be parameterized in different ways.

The celebrated Isgur-and-Karl model starts from an effective spin-spin interaction from one-gluon exchange the strength of which is adjusted to match the $\Delta(1232)$–N mass difference. This requires a rather large value for $\alpha_s$, certainly invalidating a perturbative approach. However, the one-gluon exchange is supposed to sum over many gluonic exchanges which in total carry the quantum numbers of a gluon.

Now there is an immediate problem: with this large one-gluon exchange contribution, the spin-orbit splitting becomes very large, in contrast to the experimental findings. Isgur and Karl solved this problem by assuming that the Thomas precession in the confinement potential leads to a spin-orbit splitting which cancels exactly the spin-orbit coupling originating from one-gluon exchange. This assumption allowed to reproduce the low-lying baryon resonance masses and was a break-through in the development of quark models for baryons [14]. Later, this model was further developed and refined, relativistic corrections were applied and the full energy spectrum of the relativized Hamiltonian was calculated. Results of the latest variant of this type of model can be found in [15].

An alternative model was developed by Glossman and Riska [16]. The model is based on the assumption that pions or, more generally, Goldstone bosons are exchanged between constituent quarks. The phenomenological success is impressing, in particular the low-lying $P_{11}(1440)$, the Roper resonance, is well reproduced. They emphasize the presence of parity doublets which they believe to signal chiral-symmetry restoration at high baryon masses.

The group of Metsch and Petry developed a relativistic quark model with instanton induced two-body and three-body interactions [17]. The confinement forces - which in most models are defined only in a non-relativistic frame and given as linear potential in the three-particle rest frame - have a complex Lorentz structure. They solve the Bethe-Salpeter equation by reducing it to the Salpeter equation. The parity doublets are naturally explained by instanton interactions.

Rijker, Iachello and Leviatan suggested an algebraic model of baryon resonances [18]. Their mass formula is similar to the one proposed here, but uses 10 parameters where most of them have no intuitive physical significance. On the other hand, wave functions are constructed and transition amplitudes can thus be calculated.

# 3 Symmetry considerations

## 3.1 The baryonic wave function

Symmetries play a decisive role in the classification of baryon resonances. The baryon wave function can be decomposed into a color wave function, which is antisymmetric with respect to the exchange of two quarks, the spatial and the spin-flavor wave function. The second ket in the wave function

$$|\mathrm{qqq}>=|\mathrm{colour}>_{\mathrm{A}} \cdot \underset{\mathrm{O}(6)\qquad\mathrm{SU}(6)}{|\mathrm{space};\ \mathrm{spin},\mathrm{flavour}>_{\mathrm{S}}} \tag{1}$$

has to be symmetric. The SU(6) part can be decomposed into SU(3)⊗SU(2).

## 3.2 SU(3)

In this paper, we restrict ourselves to light flavors, to *up*, *down* and *strange* quarks. The flavor wave function is then given by SU(3) and allows a decomposition

$$3 \otimes 3 \otimes 3 = 10_{\mathrm{S}} \oplus 8_{\mathrm{M}} \oplus 8_{\mathrm{M}} \oplus 1_{\mathrm{A}}, \tag{2}$$

into a decuplet which is symmetric w.r.t. the exchange of any two quarks, a singlet which is antisymmetric and two octets of mixed symmetry. The two octets have different SU(3) structures, only one of them fulfills the symmetry requirements in the total wave functions. Remember that the SU(3) multiplets contain six particle families:

| SU(3) | N | Δ | Λ | Σ | Ξ | Ω |
|---|---|---|---|---|---|---|
| 1 | no | no | yes | no | no | no |
| 8 | yes | no | yes | yes | yes | no |
| 10 | no | yes | no | yes | yes | yes |

### 3.3 SU(6)

The spin-flavor wave function can be classified according to SU(6).

$$6 \otimes\ 6 \otimes\ 6 = 56_{\rm S} \oplus\ 70_{\rm M} \oplus\ 70_{\rm M} \oplus\ 20_{\rm A} \tag{3}$$

In the ground state, the spatial wave function is symmetric, and the spin-flavor wave function has to be symmetric, too. Then, spin and flavor can both be symmetric; this is the case for the decuplet. Or spin and flavor wave function can individually have mixed symmetry, with symmetry in the combined spin-flavor wave function. This coupling represents the baryon octet. The 56-plet thus decomposes into a decuplet with spin 3/2 (four spin projections) plus an octet with spin 1/2 (two spin projections) according to

$$56 = {}^4 10\ \oplus\ {}^2 8. \tag{4}$$

The spin-flavor wave functions can also have mixed symmetry. The 70-plet can be written as

$$70 = {}^2 10\ \oplus\ {}^4 8\ \oplus\ {}^2 8\ \oplus\ {}^2 1. \tag{5}$$

Decuplet baryons, e.g. $\Delta^*$, in the 70-plet have intrinsic spin 1/2; octet baryons like excited nucleons can have spin 1/2 or 3/2. Singlet baryons with J=1/2, the $\Lambda_1$ resonances, exist only for spin-flavor wave functions of mixed symmetry. The ground state (with no orbital excitation) has no $\Lambda_1$.

The 20-plet is completely antisymmetric and requires an antisymmetric spatial wave functions. It is decomposed into an octet with spin 1/2 and a singlet with spin 3/2:

$$20 = {}^2 8\ \oplus\ {}^4 1. \tag{6}$$

### 3.4 The spatial wave function

The three-particle motion can be decomposed, in Jacobian coordinates, into two relative motions and the center-of-mass motion. The two relevant internal motions may support rotational and vibrational excitations leading to a large number of expected resonances. The spatial wave functions of mesons can be classified in the 3-dimensional rotational group O(3); the three-body motion requires O(6).

The quark dynamics can be approximated by two harmonic oscillators; to first order, harmonic-oscillator wave-functions can be use. The rotational group O(6) can be expanded into O(6)→O(3)⊗O(2), see e.g. [19]. Table 1 gives the expected multiplet structure in an O(6)⊗SU(6) classification scheme for the four lowest excitation quantum numbers $N$. With increasing $N$, an increasing number of multiplets develop. The decomposition of the orbital wave-functions results in a complicated multiplet structure of harmonic-oscillator wave-functions. It should be mentioned that some of these multiplets need two quark excitations. In the lowest 20-plet, at $N = 2$, two quarks are excited, each carrying one unit of orbital angular momentum; the two orbital angular momenta add to a total orbital angular momentum 1 and positive parity.

The ground state $N = 0$ is readily identified with the well-known octet and decuplet baryons. The first excitation band ($N = 1$) has internal orbital angular momentum L=1 ; both oscillators are excited coherently, there is one coherent excitation mode of the two oscillators. This information is comprised in the notation $3 \otimes 2_1$. The next excitation band involves several dynamical realizations. The intrinsic orbital angular momentum L can be associated with two different quarks; the vector sum of the two $l_i$ can be 0, 1 or 2 giving rise to the series $5 \otimes 1$ to $1 \otimes 1$ where the $3 \otimes 1$ is antisymmetric w.r.t. quark exchange, and the other two are symmetric. Two linearly independent coherent two-oscillator excitations exist which have mixed-symmetry spatial wave-functions. The baryon can be excited radially where the three quarks oscillate against their common center of mass. This mode is represented by $1 \otimes 1$.

With increasing $N$, the number of multiplets increases strongly; multiplets belonging to bands of up to 12 were calculated [20]. The multitude of predicted resonances escaped so far experimental observation. This is the so-called missing-resonance problem, and the basis for experimental searches for new states [21, 22].

Table 1: Multiplet-structure of harmonic oscillator wave functions. (From Hey and Kelly, Phys. Rep. 96 (1986) 71). The baryons are classified into bands that have the same number $N$ of excitation quanta. D represents the dimension of the SU(6) representation, L and P angular momentum and parity of the resonance, respectively.

| $N$ | O(6) | O(3)⊗O(2) | (D,$L_N^P$) |
|---|---|---|---|
| 0 | 1 | $1 \otimes 1$ | $(56, 0_0^+)$ |
| 1 | 6 | $3 \otimes 2_1$ | $(70, 1_1^-)$ |
| 2 | 20 | $(5+1) \otimes 2_2$ | $(70, 2_2^+)$, $(70, 0_2^+)$ |
| | | $5 \otimes 1$ | $(56, 2_2^+)$ |
| | | $3 \otimes 1$ | $(20, 1_2^+)$ |
| | 1 | $1 \otimes 1$ | $(56, 0_2^+)$ |
| 3 | 50 | $(7+3) \otimes 2_3$ | $(56, 3_3^-)$, $(20, 3_3^-)$, $(56, 1_3^-)$, $(20, 1_3^-)$ |
| | | $(7+5+3) \otimes 2_1$ | $(70, 3_3^-)$, $(70, 2_3^-)$, $(70, 1_3^-)$ |
| | 6 | $3 \otimes 2_1$ | $(70, 1_3^-)$ |
| 4 | 105 | $(9+5+1) \otimes 2_4$ | $(70, 4_4^+)$, $(70, 2_4^+)$, $(70, 0_4^+)$ |
| | | $(9+7+5+3) \otimes 2_2$ | $(70, 4_4^+)$, $(70, 3_4^+)$, $(70, 2_4^+)$, $(70, 1_4^+)$ |
| | | $(9+5+1) \otimes 1$ | $(56, 4_4^+)$, $(56, 2_4^+)$, $(56, 0_4^+)$ |
| | | $(7+5) \otimes 1$ | $(20, 3_4^+)$, $(20, 2_4^+)$ |
| | 20 | $(5+1) \otimes 1$ | $(70, 2_4^+)$, $(70, 0_4^+)$ |
| | | $3 \otimes 1$ | $(20, 1_4^+)$ |
| | | $5 \otimes 1$ | $(56, 2_4^+)$ |
| | 1 | $1 \otimes 1$ | $(56, 0_4^+)$ |

# 4 Phenomenology

## 4.1 Regge trajectories, I

It is well known that meson and baryon resonances lie on Regge trajectories, that their squared masses depend linearly on the total angular momentum J. Fig. 1 shows such a plot; Δ resonances are plotted having the lowest mass at a given total angular momentum J, with J=L+3/2 and with orbital angular momentum L even. The errors assigned will be discussed below.

The Figure also shows a meson Regge trajectory, again as a function of the total angular momentum. Light mesons with approximate isospin degeneracy and with J=L+1 are presented. The dotted line represents a fit to the meson masses taken from the PDG [8]; the error in the fit is given by the PDG errors and a second systematic error of 30 MeV added quadratically. The slope is determined to 1.142 GeV$^2$. The Δ trajectory is given by the Δ(1232) mass and the slope as determined from the meson trajectory. Obviously, mesons and Δ's have the same Regge slope. This observation is the basis for diquark models; indeed, the QCD forces between quark and antiquark are the same as those between quark and diquark.

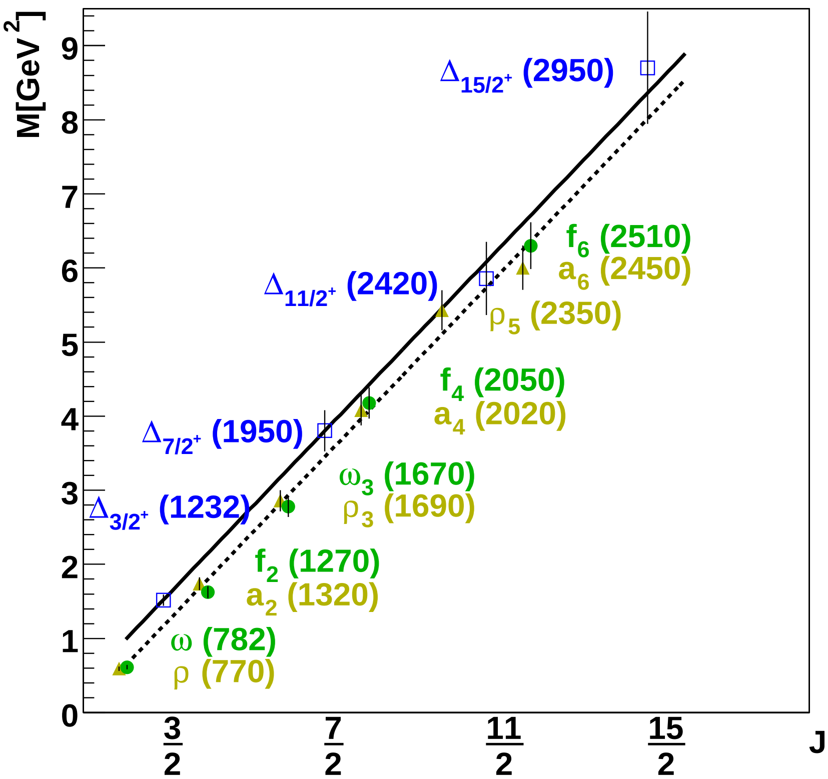


Figure 1: Δ*'s with L even and J=L+3/2. Also shown is the Regge trajectory for mesons with J=L+S.

## 4.2 Spin-orbit coupling

The starting point of our phenomenological discussion is the observation that there are no or little spin-orbit splittings in the baryon spectrum. The absence of spin-orbit splittings or, more precisely, the smallness of its contribution to meson and baryon resonances is a hotly debated subject [23]. Here, we discuss first the empirical facts and implications.

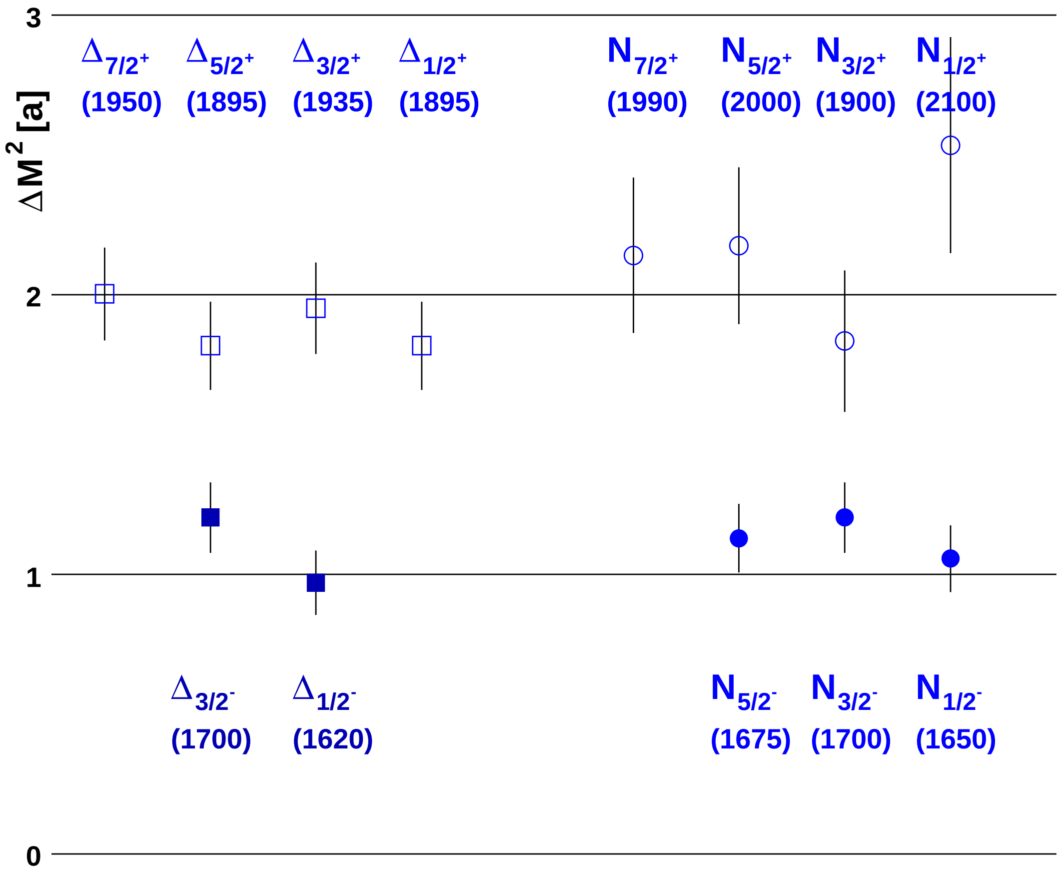


Figure 2: Δ and N resonances assigned to super-multiplets with defined spin and orbital angular momentum. Shown is the increase in mass square above the Δ(1232) [in units of a=1.142 GeV$^2$]. Upper panel: N$^*$ and Δ$^*$ with L=2 and S=3/2 coupling to $\vec{\mathrm{J}}(7/2^+, 5/2^+, 3/2^+, 1/2^+)$. Lower panel: Δ$^*$ with $\vec{\mathrm{L}}(1)+\vec{\mathrm{S}}(1/2) = \vec{\mathrm{J}}(3/2^-, 1/2^-)$ and N$^*$ with $\vec{\mathrm{L}}(1)+\vec{\mathrm{S}}(3/2) = \vec{\mathrm{J}}(5/2^-, 3/2^-, 1/2^-)$ In this and the following Figures, Δ's are represented by squares, nucleons by circles. Open symbols characterize even, full symbols odd parity.

Fig. 2 shows squared masses of (selected) positive and negative parity N and Δ resonances. The lines indicate the squared-mass values from the Regge trajectory at the first and second excitation energy. In the upper panel, there are two groups of N and Δ resonances at 1.95 GeV, two super-multiplets, with $\mathrm{J^P} = 7/2^+, 5/2^+, 3/2^+, 1/2^+$. We assign intrinsic orbital angular momentum L=2 and intrinsic spin S=3/2 to these states, with nearly vanishing spin-orbit couplings. (As discussed below, the $\mathrm{N}_{1/2^+}(2100)$ could also be the third radial excitation.) Similarly we have, at 1650 MeV, two Δ states with L=1, S=1/2 and three nucleon resonances with L=1, S=3/2. Again, no evidence for spin-orbit interactions. We conclude that spin-orbit splittings are very weak and play no decisive role for masses of baryon resonances.

## 4.3 Regge trajectories, II

We now present Figures which differ from standard Regge trajectories in choosing the orbital angular momentum L instead of J. Since spin-orbit forces are small, the orbital angular momentum is well defined. We choose L as variable since this allows us to combine baryons of positive and negative parity.

In Fig. 3 we include the $\Delta_{3/2^-}(1700)$ and the $\Delta_{7/2^-}(2220)$ to which we assign L=1, S=1/2 and L=3, S=1/2, respectively. The two resonances have the lowest mass with these quantum numbers. Their masses are fully compatible with the Regge trajectory even though the intrinsic spin of the two resonances with odd angular momentum is 1/2 whereas the other resonances have intrinsic spin 3/2. The spin-spin interaction within the mass spectrum of $\Delta$ excitations thus vanishes or is small. Since spin-orbit effects are small, we could have added the $\Delta_{1/2^-}(1620)$ at L=1.

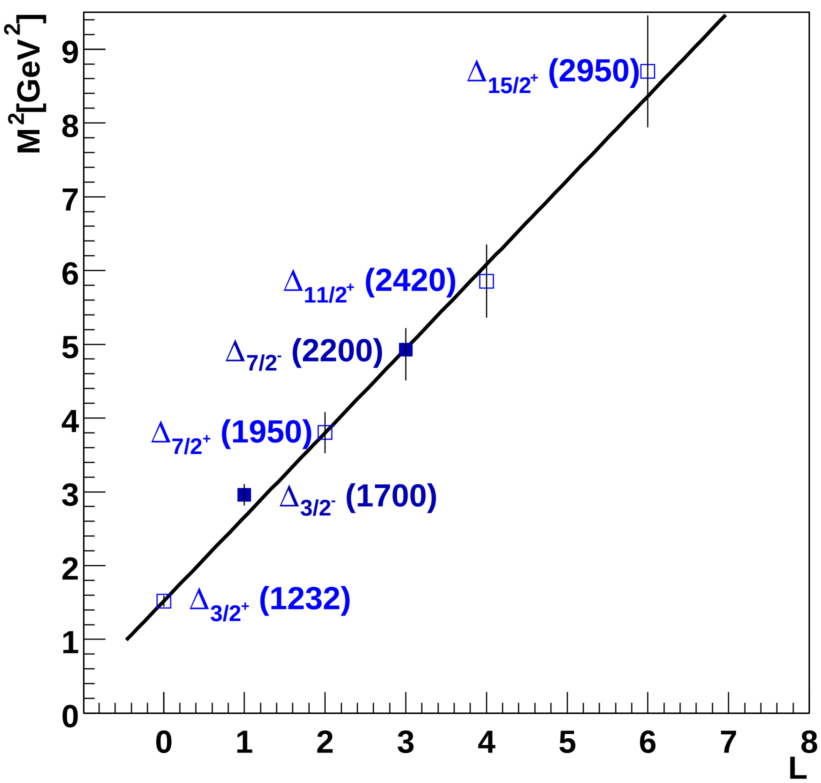


Figure 3: $\Delta^*$'s with odd L and J=L+1/2 fall on the same trajectory.

Next we turn to nucleon resonances. In Fig. 4 we have included the nucleon resonances $N_{5/2^-}(1675)$, $N_{7/2^+}(1990)$, and $N_{9/2^-}(2220)$. The $N_{5/2^-}(1675)$, $N_{3/2^-}(1700)$, and $N_{1/2^-}(1650)$ have similar masses and form a super-multiplet with L=1, S=3/2. The $N_{5/2^-}(1675)$ could also have L=3, and S=1/2 or S=3/2, the $N_{3/2^-}(1700)$ L=3, S=3/2; but then at least a $N_{7/2^-}$ in this mass range would be missing. A nucleon resonance with these quantum numbers is observed at 2190 MeV; it is accompanied by its own $N_{5/2^-}$ state (at 2200 MeV). A mixture of L=1 and L=3 both contributing to the wave function can of course not be excluded but we assume L=1 to represent the dominant part.

As seen in Fig. 2, the $N_{7/2^+}(1990)$ could be part of a super-multiplet with L=2 and S=3/2. The super-multiplet is approximately degenerate in mass with the $\Delta$ super multiplet, also having L=2 and S=3/2. The $N_{9/2^-}(2220)$ also falls onto the general Regge trajectory when we assume L=3, S=3/2 coupling to J=9/2. N and $\Delta$ resonances with a given L and with minimum mass fall on one common Regge trajectory, except nucleon resonances with intrinsic spin 1/2.

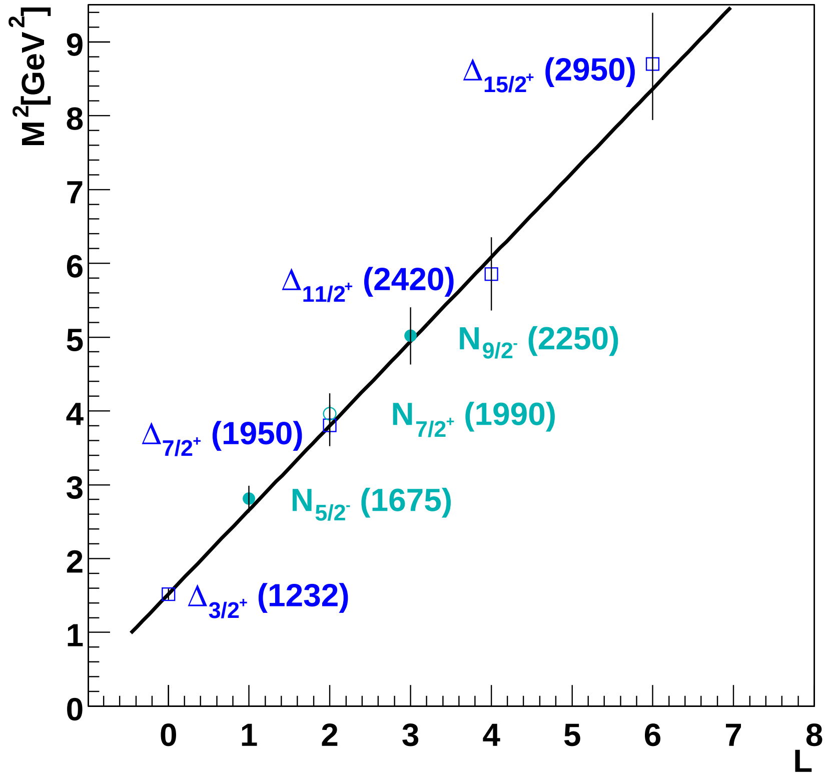


Figure 4: N*'s with intrinsic spin 3/2 fall on the same trajectory.

### 4.4 N* resonances with intrinsic spin S=1/2

So far we have not considered the nucleon and nucleon resonances with spin 1/2. In Fig. 5 we compare the squared masses of positive- and negative-parity nucleon resonances to our standard Regge trajectory. All resonances are lower in mass compared to the trajectory. The mass shifts are visualized by arrows with a length defined by the Δ(1232)-N mass splitting; for nucleons with odd angular momentum, the length of the arrow is divided by 2. The vertical lines represent the expected masses, deduced from the trajectory and a squared-mass shift calculated from

$$\mathrm{s_i} = \mathrm{M}^2_{\Delta(1232)} - \mathrm{M}^2_{\mathrm{nucleon}}. \tag{7}$$

We note that nucleons with S=1/2 are shifted in mass, nucleons with spin 3/2 not. Δ excitations have not this spin-dependent mass shift. The mass shift occurs only for baryons having spin and flavor wave-functions which are both antisymmetric w.r.t. the exchange of two quarks. This is the selection rule for instanton interactions which act only between pairs of quarks which are antisymmetric w.r.t. their exchange in spin and flavor [24]. We consider the even-odd staggering of Fig. 7 as most striking evidence for the role of instanton interactions in low-energy strong interactions.

### 4.5 Octet-decuplet splitting

The N-Δ splitting as given in eq.(7) refers not only to baryons composed of u and d quarks only. Fig. 6 compares the difference in mass square for Δ-N, Σ(1385) − Σ(1195), and Ξ(1530) − Ξ(1320). The three splittings are fully compatible and evidence the flavor-independence of the strong forces. Included are the mass square differences between the octet and singlet

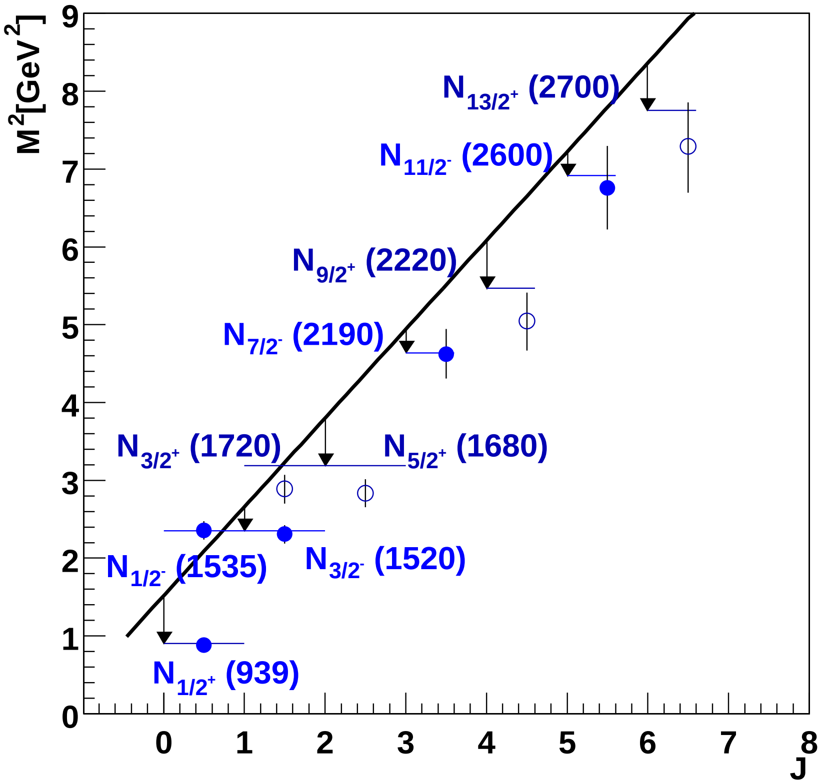


Figure 5: The N* masses (with intrinsic spin S=1/2) lie below the standard Regge trajectory. They are smaller by about 0.6 GeV$^2$ for N* in the 56-plet, and by 0.3 GeV$^2$ for N* in the 70-plet.

$\Lambda_{3/2^-}(1690) - \Lambda_{3/2^-}(1520)$ and of the $\rho - \pi$ system. Both $\Lambda$ resonances have masses which are influenced by instanton-induced interactions. The $\Lambda_{3/2^-}(1690)$ is in a 70-plet where the antisymmetric component in the wave function is reduced by a factor 1/2 compared to the nucleon. The $\Lambda_{3/2^-}(1520)$ is a SU(6) singlet state, and antisymmetric in all three quark-quark combinations. Hence this state is reduced in the squared mass by 3/2 times the value given in (7). We may thus expect, and find indeed, the same mass square difference as observed for N and $\Delta$. We assign all these splittings to instanton effects.

The $\rho - \pi$ mass-square difference is shown to argue that the same type of interaction is at work in mesons and baryons. This striking similarity is not easily understood within the frame of present-day models. It could indicate a deep symmetry between diquarks and quarks, between bosons and fermions. Strong interactions remain invariant when, in presence of a quark with spin up, a scalar diquark is replaced by a quark with spin down, or when a vector diquark is replaced by a quark with spin up [25].

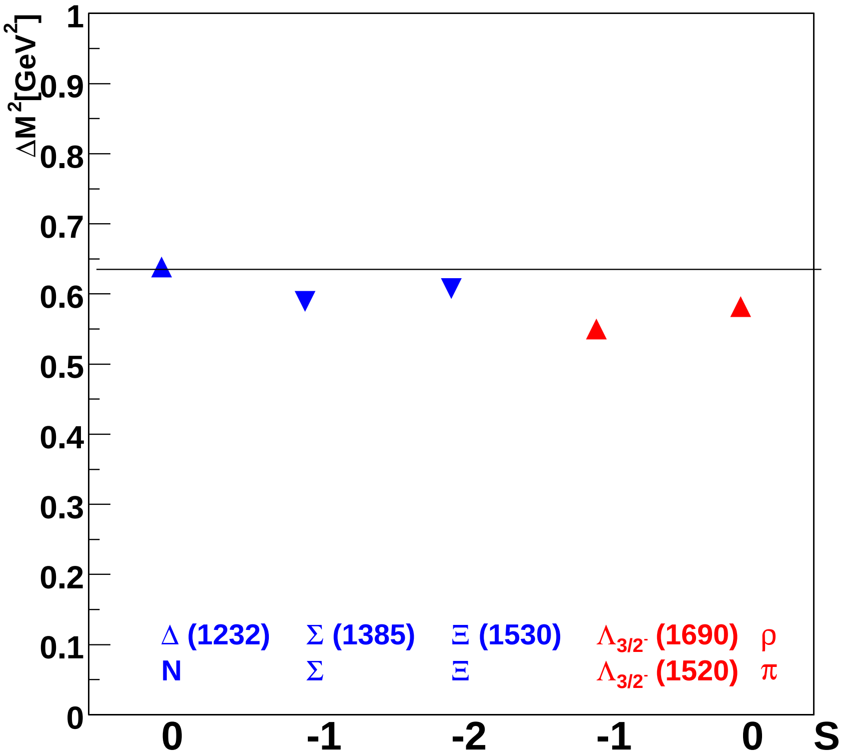


Figure 6: The difference in squared masses of octet and decuplet baryons. The difference is also shown for the two states $\Lambda_{3/2^-}(1690) - \Lambda_{3/2^-}(1520)$ where the first one belongs to the SU(3) octet, the latter to the singlet. The difference in mass square between $\rho$ and $\pi$ is of the same order of magnitude: in all cases, the same forces act.

## 4.6 Radial excitations

Some partial waves show a second resonance at a higher mass. The best known example is the Roper resonance, the $\mathrm{N}_{1/2^+}(1440)$. Its mass is rather low compared to most calculations since, in the harmonic oscillator description of baryon resonances, it is found in the second excitation band ($N = 2$).

In meson spectroscopy, radial excitations do not have a mass shift that is equivalent to the mass shift associated with two units of orbital angular momentum. The $\rho_3(1690)$ has a much higher mass than the $\rho(1450)$ (which is likely the radial excitation of the $\rho(770)$, see however ref. [26]). Based on a large number of radial excitations, Bugg concluded [27] that the mean increase in squared mass per radial excitation is $1.143 \pm 0.009\,\mathrm{GeV}^2$. This is nearly the same value we determined from the mesonic Regge trajectory. We assume that not only the slope of the Regge trajectories of mesons and baryons are the same but also the spacings between ground states and radially excited states. Thus we determine the mass of the Roper resonance by adding to the squared proton mass one unit of radial excitation energy and predict a Roper mass of 1422 MeV. The second and third radial excitations are then predicted to have masses of 1779 and 2076 MeV, respectively, to be compared with experimental candidates $\mathrm{N}_{1/2^+}(1710)$ and $\mathrm{N}_{1/2^+}(2100)$. Similarly, the first radial excitation of the $\Delta(1232)$, $\Lambda(1115)$, $\Sigma(1193)$ and $\Xi(1320)$ are predicted to have masses of 1631 (1600), 1565 (1600), 1565 (1560), and 1696 (1690) MeV; experimental values are quoted in parentheses.

There is a band of negative-parity $\Delta$ resonances, $\Delta_{5/2^-}(1930)$, $\Delta_{3/2^-}(1940)$, $\Delta_{1/2^-}(1900)$, with masses which correspond to the second excitation band. The triplet of states is naturally interpreted as first radial excitation having intrinsic orbital angular momentum 1 and spin 3/2. They are degenerate in mass with the two L=2 quartets $N_{1/2^+}(xxx)$, $N_{3/2^+}(1900)$, $N_{5/2^+}(2000)$, $N_{7/2^+}(1990)$ and $\Delta_{1/2^+}(1910)$, $\Delta_{3/2^+}(1920)$, $\Delta_{5/2^+}(1905)$, $\Delta_{7/2^+}(1950)$. Again, one unit of orbital angular momentum gives the same excitation energy as one unit in the radial quantum number. We note in passing that negative-parity $\Delta$ mesons with N=0 have a total quark spin S=1/2; the symmetry of the SU(6) wave function requires a wave function of mixed symmetry, i.e. the 70-plet. For N=1, the spatial wave function can, even for L odd, be symmetric. The SU(6) wave function can be symmetric and the $\Delta$ can be formed in a 56-plet and thus have S=3/2.

Correspondingly, we assign the $\Delta_{9/2^-}(2400)$ to L=3, S=3/2, N=1, and the $\Delta_{13/2^-}(2750)$ to L=5, S=3/2, N=1. The two states $\Delta_{7/2^-}(2200)$ and $\Delta_{5/2^-}(2350)$ should have L=3. The latter is likely a partner of the $\Delta_{9/2^-}(2400)$, the former could be the missing partner or could have S=1/2, N=0 (which we assume).

Fig. 7 collects the leading radial excitations. For clarity, the resonances are displayed as dots on the trajectory; their experimental and expected masses are compared by giving numbers.

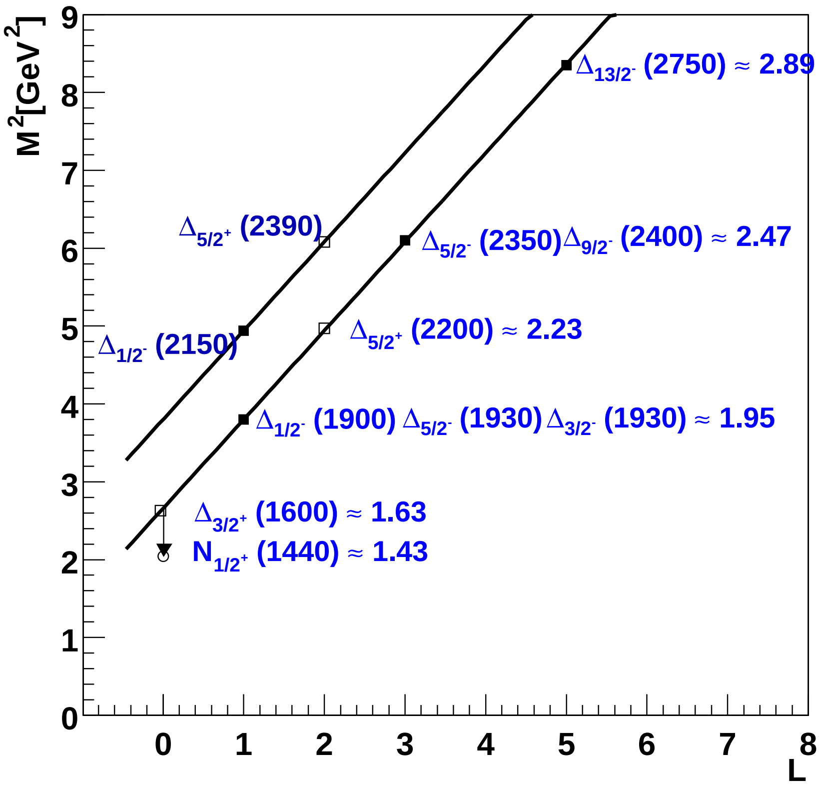


Figure 7: The masses of $\Delta$ radial excitations are compatible with Regge trajectories shifted upwards by one unit [a] in mass-square splitting. The symbols give the model-mass (also given numerically at the right side).

## 4.7 Resonances with strangeness

The mass of a baryon increases with its strangeness content. This is seen, e.g., in Fig. 8. There are small deviations from the linear interpolation when using squared masses, a linear mass interpolation gives no better agreement. We use the quadratic form, as squared masses are linear in angular momentum and squared masses are shifted by a constant value due to instanton interactions.

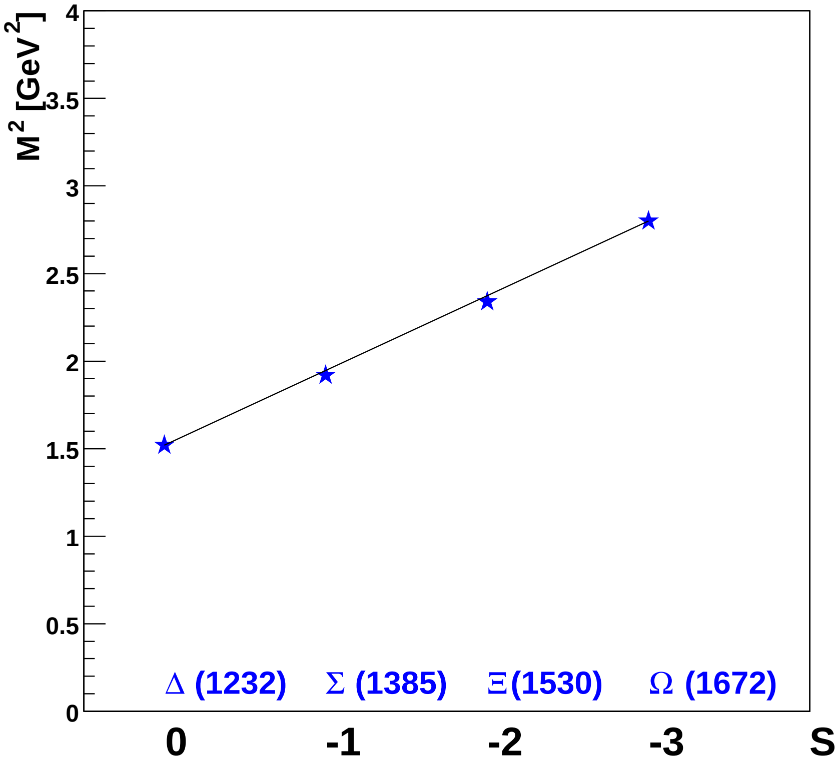


Figure 8: The squared masses for the decuplet ground state baryons as a function of their strangeness.

### 4.8 Observations and conclusions

We now recall the basic experimental observations and draw obvious conclusions from these facts.

1. The slope of the Regge trajectory for meson- and Δ-excitations is identical. Baryon resonances are quark-diquark excitations.

2. $\Delta^*$ resonances with S=1/2 and S=3/2 are on the same Regge trajectory. There is no significant spin-spin splitting due to color-magnetic interactions. Gluon exchange, often assumed to be responsible for the N-Δ splitting, should also lead to a mass shift of the $\Delta_{1/2^-}(1620)$ and $\Delta_{3/2^-}(1700)$ relative to the Regge trajectory, in the same order of magnitude as in the case of the N-Δ splitting. This is not the case. Gluon exchange is not responsible for the N-Δ splitting.

3. N and Δ resonances with spin S=3/2 lie on a common Regge trajectory. There is no genuine octet-decuplet splitting. For spin-3/2 resonances, there is no interaction associated with the SU(6) multiplet structure.

4. $N^*$'s and $\Delta^*$'s can be grouped into super-multiplets with defined orbital angular momenta L and intrinsic spin S, but different total angular momentum J. There is no significant spin-orbit ($\vec{L}\cdot\vec{S}$) interaction. This is again an argument against a large role of gluon

exchange forces (even though the spin-orbit splitting due to one-gluon exchange could be compensated by the Thomas precession in the confinement potential).

5. Octet baryons with intrinsic spin 1/2 have a shift in the squared mass. The shift is larger (by a factor 2) for even orbital angular momenta than for odd angular momenta. Wave functions of octet baryons with spin 1/2 contain a component $(\mathrm{q_1q_2 - q_2q_1})(\uparrow\downarrow - \downarrow\uparrow)$. The mass shift is proportional to this component. Instanton interactions act on quark pairs which are antisymmetric in their spin and their flavor wave function with respect to their exchange. The even-odd mass shift visible in Fig. 5 manifests the importance of instanton interactions in the baryon spectrum.

6. Daughter trajectories have the same slope as the main trajectory and an intercept which is higher by $a = 1.142\,\mathrm{GeV}^2$ per $n$, both for mesons and baryons. The similarity of the spacings between radial excitations of mesons and baryons supports again the interpretation of baryon resonances as quark-diquark excitations.

# 5 A new mass formula

## 5.1 Introduction

The observation that the mass of a baryon resonance is mostly given by its internal orbital angular momentum and the fact that spin-spin and spin-orbit splittings are small, except for instanton induced interactions, allows us to write down a simple formula which reproduces nearly all masses of baryon resonances observed so far.

## 5.2 The mass formula

The mass formula reads

$$\mathrm{M^2 = M_\Delta^2 + \frac{n_s}{3} \cdot M_s^2 + a \cdot (L + N) - s_i \cdot I_{sym}}, \tag{8}$$

where

$$\mathrm{M_s^2 = \left(M_\Omega^2 - M_\Delta^2\right)}, \qquad \mathrm{s_i = \left(M_\Delta^2 - M_N^2\right)},$$

$n_s$ the number of strange quarks in a baryon, and L the intrinsic orbital angular momentum. N is the principal quantum number (we start with N=0 for the ground state); L+2N gives the harmonic-oscillator band $N$. $\mathrm{I_{sym}}$ is the fraction of the wave function (normalized to the nucleon wave function) which is antisymmetric in spin and flavor. It is given by

$\mathrm{I_{sym}} =$ 1.0 for S=1/2 and octet in 56-plet;

$\mathrm{I_{sym}} =$ 0.5 for S=1/2 and octet in 70-plet;

$\mathrm{I_{sym}} =$ 1.5 for S=1/2 and singlet;

$\mathrm{I_{sym}} =$ 0 otherwise.

$\mathrm{M_N, M_\Delta, M_\Omega}$ are input parameters taking from PDG, $a = 1.142/\mathrm{GeV}^2$ is the Regge slope as determined from the meson spectrum.

For a quantitative comparison between data and the mass formula, masses and errors need to be defined. The Particle Data Group lists ranges of acceptable values; we use the central value for the comparison. Our error consists of two parts, of one model error of 30 MeV and one width-dependent error. The model error avoids extremely large $\chi^2$ contributions from the octet ground-state particles. The second error allows for mass shifts of resonances due to hadronic effects, like virtual decays or couplings to close-by thresholds. We estimate this effect to be of the order of one quarter of the width, and use $\Gamma/4$ as second error contribution even though we know that strong couplings to two-particle thresholds may result in much larger mass shifts. The two errors are added quadratically.

The widths of the resonances are often not well determined and, for less established baryons, no width estimate is given by the Particle Data Group. We parameterize all widths using the formula

$$\Gamma = \frac{Q}{4} \tag{9}$$

where Q is the largest available energy for hadronic decays.

## 5.3 Comparison with data

In the following Tables, we give a quantitative comparison between data and the mass formula. Overall we find $\chi^2$=117 for 97 degrees of freedom. In total, there are 103 entries in the Tables below. Three data are used to define the model: the masses of N, $\Delta$ and $\Omega$. Three states are not reproduced by the model: the $\Sigma(1480)$ with 1*, the $\Xi(1620)$ with 1*, and the $\Omega(2380)$ with 2*'s. None of them has known spin-parity. They were observed as bumps in invariant mass spectra.

First we recognize that the overall consistency of data and model is excellent. There is no free parameter used in describing the data: three baryon masses are used as input values, the slope parameter a of the Regge trajectories is taken from the meson spectrum. The $\chi^2$ achieved depends of course very critically on the error choice. A constant error contribution is needed to get the $\Lambda$ and $\Sigma$ to be compatible with one common value; there is no parameter to describe their splitting. Such a parameter would have a bad effect on high-mass states: the two $\Lambda_{1/2^-}(1800)$, $\Lambda_{5/2^-}(1830)$ have a higher mass than the triplet $\Sigma_{1/2^-}(1750)$, $\Sigma_{3/2^-}(1670)$, $\Sigma_{3/2^-}(1775)$.

Now we discuss discrepancies beyond $2\sigma$, $\chi^2 > 4$. The $\Delta_{1/2^+}(1750)$ and $\Sigma_{7/2^-}(2100)$ are 1* resonances, and the discrepancy does not need to be a failure of the model. For the $\Lambda_{1/2^+}(1600)$ and $\Lambda_{1/2^+}(1810)$, both 3* resonances, the errors given by $\Gamma/4$ are underestimated; the mass of the $\Lambda_{1/2^+}(1600)$ falls into the range from 1560 to 1700 MeV, the predicted mass is 1565 MeV. The $\Lambda_{1/2^+}(1810)$ should have a mass in the 1750 to 1850 range; the predicted value of 1895 MeV is still larger but now compatible within the model error. The two resonances $\Xi(2250)$ and $\Omega(2380)$ have no known spin-parities; it is therefore difficult to appreciate the meaning of the discrepancy. It is not excluded that in baryon resonances with two or three strange quarks, heavy-quark physics is starting to take over, that gluon exchange begins to be effective and that the extrapolation of the mass formula to $\Xi$ and $\Omega$ states is not justified. Clearly, there is not sufficient experimental information to clarify this point in a phenomenological description of data.

The $\Sigma_{3/2^-}(1580)$ and $\Sigma_{3/2^-}(1670)$ are more critical. The $\Sigma_{3/2^-}(1670)$ is a 4* resonance with a well-measured mass. It would perfectly fit, with the $\Sigma_{1/2^-}(1620)$, as $(70,^2 8)_1$ instead of $(70,^4 8)_1$ resonance. But then, the 2* state $\Sigma_{3/2^-}(1580)$ would have no slot. If we remove it, the total $\chi^2$ contribution would go down from 34.69 to 23.05 (for now 24 degrees of freedom). A 2* resonance should perhaps not be 'talked away'. But the experimental situation is certainly not clear enough to reject the model because of these two $\Sigma$ states.

Table 2: Mass spectrum of N resonances. A nucleon resonances is characterized by it $J^P$ as subscript and its nominal mass (in parenthesis). The PDG rating is given by the number of *'s. Its classification into multiplets is discussed in section (6). The PDG lists a range of acceptable values, we give the central mass (in MeV), compared to the predicted mass from eq. (8). We list the PDG range of acceptable widths Γ and compare them to eq. (9). The width parameterization is only used to estimate errors. The mass errors $\sigma$ are given by $\sigma^2 = \frac{\Gamma^2}{16} + 30^2$ where the first error allows for hadronic mass shifts in the order of 1/4 of the line width, the second one for uncertainties in the mass formula. The last column gives the $\chi^2$ contribution from the mass comparison. The $\chi^2$'s are summed up and compared to the degrees of freedom in the last column.

| Baryon | Status | $D_L$ | N | Mass | (8) | Γ | (9) | $\sigma$ | $\chi^2$ |
|---|---|---|---|---|---|---|---|---|---|
| $N_{1/2^+}(939)$ | **** | $(56,^2 8)_0$ | 0 | 939 | - | - | - | - | - |
| $N_{1/2^+}(1440)$ | **** | $(56,^2 8)_0$ | 1 | 1450 | 1423 | 250-450 | 87 | 37 | 0.53 |
| $N_{1/2^+}(1710)$ | *** | $(56,^2 8)_0$ | 2 | 1710 | 1779 | 50-250 | 176 | 53 | 1.69 |
| [1]$N_{1/2^+}(2100)$ | * | $(56,^2 8)_0$ | 2 | 2100 | 2076 | - | 251 | 70 | 0.12 |
| $N_{1/2^-}(1535)$ | **** | $(70,^2 8)_1$ | 0 | 1538 | 1530 | 100-250 | 114 | 41 | 0.04 |
| $N_{3/2^-}(1520)$ | **** | $(70,^2 8)_1$ | 0 | 1523 | 1530 | 110-135 | 114 | 41 | 0.03 |
| $N_{1/2^-}(1650)$ | **** | $(70,^4 8)_1$ | 0 | 1660 | 1631 | 145-190 | 139 | 46 | 0.4 |
| $N_{3/2^-}(1700)$ | *** | $(70,^4 8)_1$ | 0 | 1700 | 1631 | 50-150 | 139 | 46 | 2.25 |
| $N_{5/2^-}(1675)$ | **** | $(70,^4 8)_1$ | 0 | 1678 | 1631 | 140-180 | 139 | 46 | 1.04 |
| $N_{3/2^+}(1720)$ | **** | $(56,^2 8)_2$ | 0 | 1700 | 1779 | 100-200 | 176 | 53 | 2.22 |
| $N_{5/2^+}(1680)$ | **** | $(56,^2 8)_2$ | 0 | 1683 | 1779 | 120-140 | 176 | 53 | 3.28 |
| $N_{3/2^+}(1900)$ | ** | $(70,^4 8)_2$ | 0 | 1900 | 1950 | - | 219 | 62 | 0.65 |
| $N_{5/2^+}(2000)$ | ** | $(70,^4 8)_2$ | 0 | 2000 | 1950 | - | 219 | 62 | 0.65 |
| $N_{7/2^+}(1990)$ | ** | $(70,^4 8)_2$ | 0 | 1990 | 1950 | - | 219 | 62 | 0.42 |
| $N_{1/2^-}(2090)$ | * | $(70,^2 8)_1$ | 2 | 2090 | 2151 | - | 269 | 74 | 0.68 |
| $N_{3/2^-}(2080)$ | ** | $(70,^2 8)_1$ | 2 | 2080 | 2151 | - | 269 | 74 | 0.92 |
| $N_{5/2^-}(2200)$ | ** | $(70,^2 8)_3$ | 0 | 2220 | 2151 | - | 269 | 74 | 0.87 |
| $N_{7/2^-}(2190)$ | **** | $(70,^2 8)_3$ | 0 | 2150 | 2151 | 350-550 | 269 | 74 | 0 |
| $N_{9/2^-}(2250)$ | **** | $(70,^4 8)_3$ | 0 | 2240 | 2223 | 290-470 | 287 | 78 | 0.05 |
| $N_{9/2^+}(2220)$ | **** | $(56,^2 8)_4$ | 0 | 2245 | 2334 | 320-550 | 315 | 84 | 1.12 |
| $N_{11/2^-}(2600)$ | *** | $(70,^2 8)_5$ | 0 | 2650 | 2629 | 500-800 | 389 | 102 | 0.04 |
| $N_{13/2^+}(2700)$ | ** | $(56,^2 8)_6$ | 0 | 2700 | 2781 | - | 427 | 111 | 0.53 |
| | | | | | | dof: | 21 | $\sum\chi^2$: | 17.53 |

[1] Based on its mass, the $N_{1/2^+}(2100)$ is likely a radial excitation. It could also be the $(70,^4 8)_2$ $N_{1/2^+}$ state expected at 1950 MeV. The SAPHIR collaboration suggested a $N_{1/2^+}$ at 1986 MeV [28] which would, if confirmed, be a natural partner to complete the quartet of L=2, S=3/2 nucleon resonances.

Table 3: Mass spectrum of Δ resonances. See caption of Table (2).

| Baryon | Status | $D_L$ | N | Mass | (8) | Γ | (9) | σ | $\chi^2$ |
|---|---|---|---|---|---|---|---|---|---|
| $\Delta_{3/2^+}(1232)$ | **** | $(56,^4 10)_0$ | 0 | 1232 | 1232 | - | - | - | - |
| $\Delta_{3/2^+}(1600)$ | *** | $(56,^4 10)_0$ | 1 | 1625 | 1631 | 250-450 | 139 | 46 | 0.02 |
| $\Delta_{1/2^+}(1750)$ | * | $(70,^2 10)_0$ | 1 | 1750 | 1631 | - | 139 | 46 | 6.69 |
| $\Delta_{1/2^-}(1620)$ | **** | $(70,^2 10)_1$ | 0 | 1645 | 1631 | 120-180 | 139 | 46 | 0.09 |
| $\Delta_{3/2^-}(1700)$ | **** | $(70,^2 10)_1$ | 0 | 1720 | 1631 | 200-400 | 139 | 46 | 3.74 |
| $\Delta_{1/2^-}(1900)$ | ** | $(56,^4 10)_1$ | 1 | 1900 | 1950 | 140-240 | 219 | 62 | 0.65 |
| $\Delta_{3/2^-}(1940)$ | * | $(56,^4 10)_1$ | 1 | 1940 | 1950 | - | 219 | 62 | 0.03 |
| $\Delta_{5/2^-}(1930)$ | *** | $(56,^4 10)_1$ | 1 | 1945 | 1950 | 250-450 | 219 | 62 | 0.01 |
| $\Delta_{1/2^+}(1910)$ | **** | $(56,^4 10)_2$ | 0 | 1895 | 1950 | 190-270 | 219 | 62 | 0.79 |
| $\Delta_{3/2^+}(1920)$ | *** | $(56,^4 10)_2$ | 0 | 1935 | 1950 | 150-300 | 219 | 62 | 0.06 |
| $\Delta_{5/2^+}(1905)$ | **** | $(56,^4 10)_2$ | 0 | 1895 | 1950 | 280-440 | 219 | 62 | 0.79 |
| $\Delta_{7/2^+}(1950)$ | **** | $(56,^4 10)_2$ | 0 | 1950 | 1950 | 290-350 | 219 | 62 | 0 |
| $\Delta_{1/2^-}(2150)$ | * | $(70,^2 10)_1$ | 2 | 2150 | 2223 | - | 287 | 78 | 0.88 |
| $\Delta_{7/2^-}(2200)$ | * | $(70,^2 10)_3$ | 0 | 2200 | 2223 | - | 287 | 78 | 0.09 |
| [1] $\Delta_{5/2^+}(2000)$ | ** | $(70,^2 10)_2$ | 1 | 2200 | 2223 | - | 287 | 78 | 0.09 |
| $\Delta_{5/2^-}(2350)$ | * | $(56,^4 10)_3$ | 1 | 2350 | 2467 | - | 348 | 92 | 1.62 |
| $\Delta_{9/2^-}(2400)$ | ** | $(56,^4 10)_3$ | 1 | 2400 | 2467 | - | 348 | 92 | 0.53 |
| $\Delta_{7/2^+}(2390)$ | * | $(56,^4 10)_4$ | 0 | 2390 | 2467 | - | 348 | 92 | 0.7 |
| $\Delta_{9/2^+}(2300)$ | ** | $(56,^4 10)_4$ | 0 | 2300 | 2467 | - | 348 | 92 | 3.3 |
| $\Delta_{11/2^+}(2420)$ | **** | $(56,^4 10)_4$ | 0 | 2400 | 2467 | 300-500 | 348 | 92 | 0.53 |
| $\Delta_{13/2^-}(2750)$ | ** | $(56,^4 10)_5$ | 1 | 2750 | 2893 | - | 455 | 118 | 1.47 |
| $\Delta_{15/2^+}(2950)$ | ** | $(56,^4 10)_6$ | 0 | 2950 | 2893 | - | 455 | 118 | 0.23 |
| | | | | | | dof: | 21 | $\sum \chi^2$: | 22.31 |

[1] The PDG quotes two entries, at 1752 and 2200 MeV, respectively, and gives 2000 as "our estimate". We use the higher mass value for our comparison.

Table 4: Mass spectrum of Λ resonances. See caption of Table (2).

| Baryon | Status | $D_L$ | N | Mass | (8) | $\Gamma$ | (9) | $\sigma$ | $\chi^2$ |
|---|---|---|---|---|---|---|---|---|---|
| $\Lambda_{1/2^+}(1115)$ | **** | $(56,^2 8)_0$ | 0 | 1116 | 1144 | - | - | 30 | 0.87 |
| $\Lambda_{1/2^+}(1600)$ | *** | $(56,^2 8)_0$ | 1 | 1630 | 1565 | 50-250 | 32 | 31 | 4.4 |
| $\Lambda_{1/2^+}(1810)$ | *** | $(56,^2 8)_0$ | 2 | 1800 | 1895 | 50-250 | 115 | 42 | 5.12 |
| $\Lambda_{1/2^-}(1405)$ | **** | $(70,^2 1)_1$ | 0 | 1407 | 1460 | 50 | 6 | 30 | 3.12 |
| $\Lambda_{3/2^-}(1520)$ | **** | $(70,^2 1)_1$ | 0 | 1520 | 1460 | 16 | 6 | 30 | 4 |
| $\Lambda_{1/2^-}(1670)$ | **** | $(70,^2 8)_1$ | 0 | 1670 | 1664 | 25-50 | 57 | 33 | 0.03 |
| $\Lambda_{3/2^-}(1690)$ | **** | $(70,^2 8)_1$ | 0 | 1690 | 1664 | 50-70 | 57 | 33 | 0.62 |
| $\Lambda_{1/2^-}(1800)$ | *** | $(70,^4 8)_1$ | 0 | 1785 | 1757 | 200-400 | 80 | 36 | 0.6 |
| $\Lambda_{5/2^-}(1830)$ | **** | $(70,^4 8)_1$ | 0 | 1820 | 1757 | 60-110 | 80 | 36 | 3.06 |
| $\Lambda_{3/2^+}(1890)$ | **** | $(56,^2 8)_2$ | 0 | 1880 | 1895 | 60-200 | 115 | 42 | 0.13 |
| $\Lambda_{5/2^+}(1820)$ | **** | $(56,^2 8)_2$ | 0 | 1820 | 1895 | 70-90 | 115 | 42 | 3.19 |
| $\Lambda(2000)$ | * | $(70,^4 8)_2$ | 0 | 2000 | 2056 | - | 155 | 49 | 1.31 |
| $\Lambda_{5/2^+}(2110)$ | *** | $(70,^4 8)_2$ | 0 | 2115 | 2056 | 150-250 | 155 | 49 | 1.45 |
| $\Lambda_{7/2^+}(2020)$ | * | $(70,^4 8)_2$ | 0 | 2020 | 2056 | - | 155 | 49 | 0.54 |
| $\Lambda_{7/2^-}(2100)$ | **** | $(70,^2 1)_3$ | 0 | 2100 | 2101 | 100-250 | 166 | 51 | 0 |
| $\Lambda_{3/2^-}(2325)$ | * | $(70,^2 8)_1$ | 2 | 2325 | 2248 | - | 203 | 59 | 1.7 |
| $\Lambda_{9/2^+}(2350)$ | *** | $(56,^2 8)_4$ | 0 | 2355 | 2424 | 100-250 | 247 | 69 | 1 |
| $\Lambda(2585)$ | ** | $(70,^4 8)_2$ | 0 | 2585 | 2551 | - | 279 | 76 | 0.2 |
| | | | | | | dof: | 18 | $\sum \chi^2$: | 31.34 |

Table 5: Mass spectrum of $\Sigma$ resonances. See caption of Table (2).

| Baryon | Status | $D_L$ | N | Mass | (8) | $\Gamma$ | (9) | $\sigma$ | $\chi^2$ |
|---|---|---|---|---|---|---|---|---|---|
| $\Sigma_{1/2^+}(1193)$ | **** | $(56,^2 8)_0$ | 0 | 1193 | 1144 | - | - | 30 | 2.67 |
| $\Sigma_{3/2^+}(1385)$ | **** | $(56,^4 10)_0$ | 0 | 1384 | 1394 | - | - | 30 | 0.11 |
| $\Sigma(1480)$ | * | | | | | | | | |
| $\Sigma(1560)$ | ** | $(56,^2 8)_0$ | 1 | 1560 | 1565 | - | 32 | 31 | 0.03 |
| $\Sigma_{1/2^+}(1660)$ | *** | $(70,^2 8)_0$ | 1 | 1660 | 1664 | 40-200 | 57 | 33 | 0.01 |
| $\Sigma_{1/2^+}(1770)$ | * | $(70,^2 10)_0$ | 1 | 1770 | 1757 | - | 80 | 36 | 0.13 |
| $\Sigma_{1/2^+}(1880)$ | ** | $(56,^2 8)_0$ | 2 | 1880 | 1895 | - | 115 | 42 | 0.13 |
| $\Sigma_{1/2^-}(1620)$ | ** | $(70,^2 8)_1$ | 0 | 1620 | 1664 | - | 57 | 33 | 1.78 |
| $\Sigma_{3/2^-}(1580)$ | ** | $(70,^2 8)_1$ | 0 | 1580 | 1664 | - | 57 | 33 | 6.48 |
| $\Sigma(1690)$ | ** | $(70,^2 10)_1$ | 0 | 1690 | 1757 | - | 80 | 36 | 3.46 |
| $\Sigma_{1/2^-}(1750)$ | *** | $(70,^4 8)_1$ | 0 | 1765 | 1757 | 60-160 | 80 | 36 | 0.05 |
| $\Sigma_{3/2^-}(1670)$ | **** | $(70,^4 8)_1$ | 0 | 1675 | 1757 | 40-80 | 80 | 36 | 5.19 |
| $\Sigma_{5/2^-}(1775)$ | **** | $(70,^4 8)_1$ | 0 | 1775 | 1757 | 105-135 | 80 | 36 | 0.25 |
| $\Sigma_{1/2^-}(2000)$ | * | $(70,^2 8)_1$ | 1 | 2000 | 1977 | - | 135 | 45 | 0.26 |
| $\Sigma_{3/2^-}(1940)$ | *** | $(70,^2 8)_1$ | 1 | 1925 | 1977 | 150-300 | 135 | 45 | 1.34 |
| $\Sigma_{3/2^+}(1840)$ | * | $(56,^2 8)_2$ | 0 | 1840 | 1895 | - | 115 | 42 | 1.71 |
| $\Sigma_{5/2^+}(1915)$ | **** | $(56,^2 8)_2$ | 0 | 1918 | 1895 | 80-160 | 115 | 42 | 0.3 |
| [1]$\Sigma_{3/2^+}(2080)$ | ** | $(56,^4 10)_2$ | 0 | 2080 | 2056 | - | 155 | 49 | 0.24 |
| [1]$\Sigma_{5/2^+}(2070)$ | * | $(56,^4 10)_2$ | 0 | 2070 | 2056 | - | 155 | 49 | 0.06 |
| [1]$\Sigma_{7/2^+}(2030)$ | **** | $(56,^4 10)_2$ | 0 | 2033 | 2056 | 150-200 | 155 | 49 | 0.22 |
| $\Sigma(2250)$ | *** | $(70,^2 8)_3$ | 0 | 2245 | 2248 | 60-150 | 203 | 59 | 0 |
| $\Sigma_{7/2^-}(2100)$ | * | $(70,^2 8)_3$ | 0 | 2100 | 2248 | - | 203 | 59 | 6.29 |
| $\Sigma(2455)$ | ** | $(56,^2 8)_4$ | 0 | 2455 | 2424 | - | 247 | 69 | 0.2 |
| $\Sigma(2620)$ | ** | $(70,^2 8)_5$ | 0 | 2620 | 2708 | - | 318 | 85 | 1.07 |
| $\Sigma(3000)$ | * | $(56,^2 8)_6$ | 0 | 3000 | 2857 | - | 355 | 94 | 2.31 |
| $\Sigma(3170)$ | * | $(70,^2 8)_7$ | 0 | 3170 | 3102 | - | 416 | 108 | 0.4 |
| | | | | | | dof: | 25 | $\sum \chi^2$: | 34.69 |

[1] These three resonances, and the missing $\Sigma_{1/2^+}$, can belong to the octet or to the decuplet; the mass formula (8) predicts identical masses.

Table 6: Mass spectrum of Ξ resonances. See caption of Table (2).

| Baryon | Status | $D_L$ | N | Mass | (8) | $\Gamma$ | (9) | $\sigma$ | $\chi^2$ |
|---|---|---|---|---|---|---|---|---|---|
| $\Xi_{1/2^+}(1320)$ | **** | $(56,^2 8)_0$ | 0 | 1315 | 1317 | - | - | 30 | 0 |
| $\Xi_{3/2^+}(1530)$ | **** | $(56,^4 10)_0$ | 0 | 1532 | 1540 | 9 | - | 30 | 0.07 |
| $\Xi(1620)$ | * | | | 1620 | | | | | |
| $\Xi(1690)$ | *** | $(56,^2 8)_0$ | 1 | 1690 | 1696 | <30 | 21 | 30 | 0.04 |
| $\Xi_{3/2^-}(1820)$ | *** | $(70,^2 8)_1$ | 0 | 1823 | 1787 | 14-39 | 43 | 32 | 1.27 |
| $\Xi(1950)$ | *** | $(56,^2 8)_2$ | 0 | 1950 | 2004 | 40-80 | 98 | 39 | 1.92 |
| $\Xi(2030)$ | *** | $(56,^2 8)_2$ | 0 | 2025 | 2004 | 15-35 | 98 | 39 | 0.29 |
| $\Xi(2120)$ | * | $(56,^4 10)_2$ | 0 | 2120 | 2157 | - | 136 | 45 | 0.68 |
| $\Xi(2250)$ | ** | $(56,^4 10)_2$ | 0 | 2250 | 2157 | - | 136 | 45 | 4.27 |
| $\Xi(2370)$ | ** | $(70,^2 8)_3$ | 0 | 2370 | 2340 | - | 182 | 55 | 0.3 |
| $\Xi(2500)$ | * | $(56,^2 8)_4$ | 0 | 2500 | 2510 | - | 224 | 64 | 0.02 |
| | | | | | | dof: | 10 | $\sum\chi^2$: | 8.86 |

Table 7: Mass spectrum of Ω resonances. See caption of Table (2).

| Baryon | Status | $D_L$ | N | Mass | (8) | $\Gamma$ | (9) | $\sigma$ | $\chi^2$ |
|---|---|---|---|---|---|---|---|---|---|
| $\Omega_{3/2^+}(1672)$ | **** | $(56,^4 10)_0$ | 0 | 1672 | - | - | - | - | - |
| $\Omega(2250)$ | **** | $(56,^4 10)_2$ | 0 | 2252 | 2254 | 37-73 | 77 | 36 | 0 |
| $\Omega(2380)$ | ** | - | - | 2380 | - | - | - | - | - |
| $\Omega(2470)$ | ** | $(70,^2 10)_3$ | 0 | 2474 | 2495 | 39-105 | 137 | 46 | 0.21 |
| | | | | | | dof: | 2 | $\sum\chi^2$: | 0.21 |

# 6 Multiplet structure of observed and missing resonances

## 6.1 Missing resonances

We now discuss how baryon resonances can be assigned to given multiplets. The emphasis of this discussion will be to identify the nature of the so-called *missing* resonances. There are different reasons for resonances not to be observed, and theoretical guidance (or prejudices) are needed to understand why a particular resonance has not been observed. We distinguish between different classes of *missing* resonances:

1. *Trivially missing resonances*, resonances which are expected to exist in a model-independent way. E.g., there is no known $\Omega_{3/2^-}$ state even though nobody will doubt that it would be discovered in an appropriate experiment.

2. There are *hidden* resonances, resonances with identical quantum numbers having, according to our mass formula (8), the same mass but differing in their internal spin-flavor structure. From the nucleon resonances at 1950 MeV we know that there is a $^{4}8$ multiplet at about 1950 MeV. They belong to a 70-plet. We must therefore expect a $\Delta$ spin doublet $^{2}10$. According to (8) the $^{2}10$ doublet has the same mass as the spin quartet $^{4}10$. There are hence two $\Delta_{3/2^+}(1950)$ and two $\Delta_{5/2^+}(1950)$ states expected. The second radial excitation of the $\Delta(1232)$ is also expected as $\Delta_{3/2^+}(1950)$. A careful high-statistics study of several decay modes could possibly reveal that more than one resonance contributes; at the present level of experiments they are unobservable. We call these missing resonances *hidden* resonances.

3. Above 2.5 GeV, only stretched resonances are observed, with spin and orbital angular momentum parallel. Nucleon resonances have spin 1/2 and J=L+1/2; $\Delta$ excitations prefer S=3/2 and J=L+3/2. Since S=3/2 is forbidden for N=0, $\Delta$'s with S=3/2 must have one unit of radial excitation. Clearly, other $(\vec{\mathrm{L}}\cdot\vec{\mathrm{S}})$ couplings are possible and two nucleon- or four $\Delta$-resonances with approximately the same mass but different J could exist. These states are solutions of the Hamiltonian; but this does not guarantee that they are realized dynamically as stable rotations. A match box has three axes of rotation, dynamically realized are only those around the axis of minimal and maximal moment of inertia. We call resonances expected as solutions of the Hamiltonian but not realized dynamically *missing resonances*. Of course, *hidden* resonances can also be suppressed dynamically and thus belong to the class of *missing resonances*.

## 6.2 Ground states

The ground states of octet and decuplet baryons are, of course, all experimentally well established. The N, $\Delta(1232)$, and $\Omega$ masses are used as input parameters. There is no parameter allowing for a $\Lambda(1115)-\Sigma(1193)$ splitting which amounts to 77 MeV. The model predicts 1144 MeV. For resonances, there is no general mass enhancement of $\Sigma^*$'s compared to $\Lambda^*$'s. The $\Sigma(1385)$ and $\Xi(1530)$ decuplet ground states are rather well reproduced in the model.

## 6.3 Radial excitations of the ground states

We have seen that there is a sequence of spin $1/2^+$ nucleon and spin $3/2^+$ $\Delta$ resonances which can be assigned to radial excitations of the respective ground state. These, and corresponding states for the hyperons, are collected in Table 8. In the harmonic-oscillator representation of radially excited states, there are two types of resonances expected in the second band, having $\mathrm{N}_{1/2^+}$ quantum numbers with $(\mathrm{D},\mathrm{L}^{\mathrm{P}}_{\mathrm{N}}) = (56,0^+_2)$ and $(70,0^+_2)$ SU(6) wave functions. Decuplet baryons in the 56- and 70-plet are expected to be mass-degenerate; octet baryons in 56 or 70 feel different instanton-induced interactions. The members of the 70-plet would be $\mathrm{N}_{1/2^+}(1530)$, $\Lambda_{1/2^+}(1660)$, $\Sigma_{1/2^+}(1660)$ and $\Xi_{1/2^+}(1787)$. In the decuplet we expect $\Delta_{1/2^+}(1631)$, $\Delta_{1/2^+}(1950)$, and a $\Sigma_{1/2^+}(1757)$. One radially excited $\Xi$ state is observed, further states and $\Omega$ resonances are *trivially* missing.

We note that N and $\Delta$ radial excitations are mostly compatible with an assignment to a 56-plet and not to 70-plets. The 1* $\Delta_{1/2^+}(1750)$ is an exception of this rule. It is not clear if there are really two resonances $\Sigma_{1/2^+}(1660)$ and $\Sigma_{1/2^+}(1770)$ but both need to be assigned to a radial excitation in a 70-plet. Hence we believe that the 70-plet is needed to complete the spectrum of radially excited states. The absence of radially excited nucleon states in the 70-plet could reflect a dynamical suppression: the 70-plet could possibly be formed only in the case of unequal quark masses. This conjecture would require the $\Delta_{1/2^+}(1750)$ not to exist and remove the largest single $\chi^2$ contribution. This question certainly needs theoretical study. For the ground states, the 70-plet is of course forbidden due to symmetry reasons.

## 6.4 Resonances in the first harmonic-oscillator band

The lowest orbital-angular-momentum excitations have L=1; in SU(6) a 70-plet is expected which can be decomposed into SU(3) multiplets:

$$70 = {}^{2}10 \,\oplus\, {}^{4}8 \,\oplus\, {}^{2}8 \,\oplus\, {}^{2}1.$$

**The spin-3/2 resonances:** Spin-3/2 plus L=1 forms a spin-triplet of resonances, with quantum numbers $1/2^-$, $3/2^-$, and $5/2^-$. The three states $\mathrm{N}_{1/2^-}(1650)$, $\mathrm{N}_{3/2^-}(1700)$, $\mathrm{N}_{5/2^-}(1675)$ obviously match our expectation, as well as the $\Sigma_{1/2^-}(1750)$, $\Sigma_{3/2^-}(1670)$, $\Sigma_{5/2^-}(1775)$ resonances. (We remind the reader that the $\Sigma_{3/2^-}(1670)$ could also be the partner of the $\Sigma_{1/2^-}(1620)$, provided the $\Sigma_{3/2^-}(1580)$ does not exist.) In the $\Lambda$ sector, the $\Lambda_{1/2^-}(1800)$ and $\Lambda_{5/2^-}(1830)$ can be assigned to the spin-3/2 octet states, the $\Lambda_{3/2^-}$ is missing.

There is only one corresponding $\Xi$ resonance, the $\Xi_{3/2^-}(1820)$. It is assigned to the octet but it could also belong to the decuplet where it should have a mass of 1874 MeV. The masses of experimentally known $\Omega$ resonances are incompatible with the calculated masses of (L=1,N=0) or (L=0,N=1) resonances.

We expect three spin-doublets: in SU(3) singlet, octet, and decuplet.

**Singlet:** The two states $\Lambda_{1/2^-}(1405)$, $\Lambda_{3/2^-}(1520)$ are very low in mass. From (8) we predict 1460 MeV for the singlet L=1 states, in reasonable agreement with the experimental values. The mass formula assumes that instanton interactions lead to a mass shift in the singlet - with all three quarks antisymmetric w.r.t. the exchange of two quarks - which is three times larger than for the octet state where only one quark pair is antisymmetric w.r.t. exchange of the two quarks.

Table 8: SU(3) octet and decuplet radial excitations of baryon resonances. The 6th and the last column give masses as calculated from eq. (8).

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| L=0 | N=0 | 56 | $^2 8$ | $N_{1/2^+}(939)$ | 939 | $^4 10$ | $\Delta_{3/2^+}(1232)$ | 1232 |
| L=0 | N=1 | 56 | $^2 8$ | $N_{1/2^+}(1440)$ | 1422 | $^4 10$ | $\Delta_{3/2^+}(1600)$ | 1631 |
| L=0 | N=1 | 70 | $^2 8$ | | 1530 | $^2 10$ | $\Delta_{1/2^+}(1750)$ | 1631 |
| L=0 | N=2 | 56 | $^2 8$ | $N_{1/2^+}(1710)$ | 1779 | $^4 10$ | $\Delta_{3/2^+}(1920)$ | 1950 |
| L=0 | N=3 | 56 | $^2 8$ | $N_{1/2^+}(2100)$ | 2076 | $^4 10$ | | 2223 |
| L=0 | N=0 | 56 | $^2 8$ | $\Lambda_{1/2^+}(1115)$ | 1143 | | | |
| L=0 | N=1 | 56 | $^2 8$ | $\Lambda_{1/2^+}(1600)$ | 1565 | | | |
| L=0 | N=2 | 56 | $^2 8$ | $\Lambda_{1/2^+}(1810)$ | 1895 | | | |
| L=0 | N=0 | 56 | $^2 8$ | $\Sigma_{1/2^+}(1193)$ | 1143 | $^4 10$ | $\Sigma_{3/2^+}(1385)$ | 1394 |
| L=0 | N=1 | 56 | $^2 8$ | $\Sigma_{1/2^+}(1560)$ | 1565 | $^4 10$ | | 1757 |
| L=0 | N=1 | 70 | $^2 8$ | $\Sigma_{1/2^+}(1660)$ | 1664 | $^2 10$ | $\Sigma_{1/2^+}(1770)$ | 1757 |
| L=0 | N=2 | 56 | $^2 8$ | $\Sigma_{1/2^+}(1880)$ | 1895 | $^2 10$ | | 2056 |
| L=0 | N=0 | 56 | $^2 8$ | $\Xi_{1/2^+}(1320)$ | 1317 | $^4 10$ | $\Xi_{3/2^+}(1530)$ | 1540 |
| L=0 | N=1 | 56 | $^2 8$ | $\Xi(1690)$ | 1696 | $^4 10$ | | 1869 |
| L=0 | N=0 | 56 | | | | $^4 10$ | $\Omega_{3/2^+}(1672)$ | 1672 |
| L=0 | N=1 | 56 | | | | $^4 10$ | | 1984 |

**Octet:** The states $N_{1/2^-}(1535)$, $N_{3/2^-}(1520)$ (expected mass 1530 MeV); $\Lambda_{1/2^-}(1670)$, $\Lambda_{3/2^-}(1690)$ and $\Sigma_{1/2^-}(1620)$, $\Sigma_{3/2^-}(1580)$ or $\Sigma_{3/2^-}(1670)$ (expected mass 1664 MeV); and $\Xi_{3/2^-}(1820)$ (expected mass 1787 MeV) fill this slot; only the $\Xi_{1/2^-}$ is missing.

**Decuplet:** In case of $\Delta$ and $\Omega$ resonances, the assignment is conceptually easy, even though there is no candidate for the two $\Omega$ resonances. The $\Delta$ doublet is observed at $\Delta_{1/2^-}(1620)$, $\Delta_{3/2^-}(1700)$ and expected at 1631 MeV.

In case of the $\Sigma$ and $\Xi$, which contribute to the octet and the decuplet, we expect two additional states. We have combined the two states $\Sigma_{1/2^-}(1750)$, $\Sigma_{3/2^-}(1670)$ with the $\Sigma_{5/2^-}(1775)$ to form a triplet of states, expected from the $^4 8$ part of the 70-plet. Now, the two $^2 10$ states - expected to have the same mass - are missing. Similarly, the $\Xi_{3/2^-}(1820)$ and the unobserved $\Xi_{1/2^-}$ could belong to the octet and to the decuplet. As octet states they have spin 3/2, as decuplet states spin 1/2. In both cases, they do not undergo instanton interactions and the predicted masses are the same. These are *hidden* resonances in our nomenclature. Experimentally, there are indications for a doubling of states with identical quantum numbers but different decay modes. See the comment of the Particle Data Group in the full listing for $\Sigma(1670)$ and $\Sigma(1690)$ bumps.

## 6.5 Resonances in the second harmonic-oscillator band

In the second band of the harmonic-oscillator the following multiplets are expected:

$(\mathbf{56}, \mathbf{0}^+_\mathbf{2})$ **and** $(\mathbf{70}, \mathbf{0}^+_\mathbf{2})$**:** The scalar excitations were discussed in section 6.3. Both multiplets contain entries. Possibly, there is a selection rule preventing $(70, 0^+_2)$ states for three identical quark masses.

$(\mathbf{20}, \mathbf{1}^+_\mathbf{2})$**:** The multiplet has no component which is antisymmetric in spin and flavor. The $N^*$ resonances are hidden behind the $N_{5/2^+}(2000)$ and $N_{3/2^+}(1900)$, one of the $^41$ $\Lambda$ states behind the $\Lambda_{5/2^+}(2020)$, the other two $\Lambda_{3/2^+}$ and $\Lambda_{1/2^+}$ are expected also at 2056 MeV. In these states, both harmonic oscillators are excited, each with one unit of angular momentum. (The two angular momenta couple to a total orbital angular momentum 1.) These states are difficult to excite but could be narrow. So far, we have no evidence that resonances are formed in $(20, 1^+_2)$ and we do not discuss such states further down.

$(\mathbf{56}, \mathbf{2}^+_\mathbf{2})$ **and** $(\mathbf{70}, \mathbf{2}^+_\mathbf{2})$**:** From the $(56, 2^+_2)$ multiplet, we expect a spin-1/2 octet and a spin-3/2 decuplet. The octet states with L=2 and spin 1/2 can be identified with the $N_{3/2^+}(1720)$ and $N_{5/2^+}(1680)$, the $\Lambda_{3/2^+}(1890)$ and $\Lambda_{5/2^+}(1820)$ and the $\Sigma_{3/2^+}(1840)$ and $\Sigma_{5/2^+}(1915)$. The two states, $\Xi(1950)$ and $\Xi(2030)$, have the expected mass (2004 MeV) but spin and parity are not known. Decuplet states with spin 3/2 are obviously the four resonances $\Delta_{1/2^+}(1910)$, $\Delta_{3/2^+}(1920)$, $\Delta_{5/2^+}(1905)$, and $\Delta_{7/2^+}(1950)$, all expected at 1950 MeV. The $\Sigma_{7/2^+}(2030)$ state which must have S=3/2; the $^2\Sigma_{3/2^+}(2080)$, $^2\Sigma_{5/2^+}(2070)$ are natural partners in this super-multiplet. The $\Xi(2120)$ and $\Xi(2250)$ have masses which are not incompatible with the expected 2157 MeV. There is also the $\Omega(2470)$ resonance with a mass compatible with an L=2 excitation.

The $\Sigma$ and $\Xi$ states assigned to the decuplet in 56-plet with spin 3/2 could also belong to the octet with spin 3/2 in the 70-plet. As spin=3/2 states their masses are not affected by instanton-induced interactions; these super-multiplets are predicted to coincide in mass, the super-multiplets are *hidden*. The question if there are states belonging to $^48$ multiplets can only be decided for baryons which contribute only to the octet and not to the decuplet.

There is one $N_{7/2^+}$ and one $\Lambda_{7/2^+}$ state at about 2 GeV; their masses are too low to assign an internal angular momentum L=4 to these states. Then, they have to have spin S=3/2 and have to belong to the 70-plet. These two states are very important; they entail the existence of many further states. In the N sector, a nearly complete quartet can be made up, the $N_{3/2^+}(1900)$, $N_{5/2^+}(2000)$, $N_{7/2^+}(1990)$ (with the fourth member possibly seen by [28]). The $\Lambda_{5/2^+}(2110)$ is likely the companion of the $\Lambda_{7/2^+}(2020)$. Hence there are L=2 states belonging to the 70-plet.

In the 70-plet we expect not only the octet with spin 3/2 but also spin 1/2 multiplets, in singlet, octet and decuplet. As mentioned above, the decuplet-spin-1/2 states are mass-degenerate with the spin-3/2 states and are hence difficult to establish. But the singlet and octet states should be observable.

We should expect a doublet of SU(3) singlet states with L=2 and S=1/2. These states undergo strong instanton interactions, with a symmetry factor in eq. (8) of 1.5 instead of 1 (for the octet $\Lambda$'s). The mass is calculated to 1809 MeV while the octet states are expected at 1895 MeV. We note that the $\Lambda_{5/2^+}(1820)$ fits excellently to the mass prediction for the singlet state. However, to claim that the $\Lambda_{5/2^+}(1820)$ belongs to the singlet system - together with the $\Lambda_{1/2^-}(1405)$, $\Lambda_{3/2^-}(1520)$, and $\Lambda_{7/2^-}(2100)$ - is certainly not justified without observing a doublet structure.

Table 9: Mass spectrum of observed and missing N resonances. Nominal masses are from the PDG listings (in MeV). The last column gives masses from eq. (8).

| | | M |
|---|---|---|
| L=2 N=0 56 $^2 8$ | $N_{3/2^+}(1720)$, $N_{5/2^+}(1680)$ | 1779 |
| L=2 N=0 56 $^2 8$ | $\Lambda_{3/2^+}(1890)$, $\Lambda_{5/2^+}(1820)$ | 1895 |
| L=2 N=0 56 $^2 8$ | $\Sigma_{3/2^+}(1840)$, $\Sigma_{5/2^+}(1915)$ | 1895 |
| L=2 N=0 56 $^2 8$ | $\Xi_{3/2^+}(xxx)$, $\Xi_{5/2^+}(1950)$ | 2005 |
| L=2 N=0 56 $^4 10$ | $\Delta_{1/2^+}(1910)$, $^1\Delta_{3/2^+}(1920)$, $^1\Delta_{5/2^+}(1905)$, $\Delta_{7/2^+}(1950)$ | 1950 |
| L=2 N=0 56 $^4 10$ | $^2\Sigma_{1/2^+}(xxx)$,$^2\Sigma_{3/2^+}(2080)$, $^2\Sigma_{5/2^+}(2070)$, $^2\Sigma_{7/2^+}(2030)$ | 2056 |
| L=2 N=0 56 $^4 10$ | $\Xi_{1/2^+}(xxx)$, $\Xi_{3/2^+}(xxx)$, $\Xi_{5/2^+}(xxx)$, $\Xi_{7/2^+}(2120)$ | 2157 |
| L=2 N=0 70 $^2 8$ | $N_{3/2^+}(xxx)$, $N_{5/2^+}(xxx)$ | 1866 |
| L=2 N=0 70 $^4 8$ | $N_{1/2^+}(xxx)$, $N_{3/2^+}(1900)$, $N_{5/2^+}(2000)$, $N_{7/2^+}(1990)$ | 1950 |
| L=2 N=0 70 $^2 10$ | $^1\Delta_{3/2^+}(xxx)$,$^1\Delta_{5/2^+}(xxx)$ | 1950 |
| L=2 N=0 70 $^2 8$ | $\Lambda_{3/2^+}(xxx)$, $\Lambda_{5/2^+}(xxx)$ | 1977 |
| L=2 N=0 70 $^4 8$ | $\Lambda_{1/2^+}(xxx)$, $\Lambda(2000)$, $\Lambda_{5/2^+}(2110)$, $\Lambda_{7/2^+}(2020)$ | 2056 |
| L=2 N=0 70 $^2 8$ | $^2\Sigma_{3/2^+}(xxx)$, $^2\Sigma_{5/2^+}(xxx)$, | 1977 |
| L=2 N=0 70 $^4 8$ | $^2\Sigma_{1/2^+}(xxx)$,$^2\Sigma_{3/2^+}(xxx)$, $^2\Sigma_{5/2^+}(xxx)$, $^2\Sigma_{7/2^+}(xxx)$ | 2056 |
| L=2 N=0 70 $^2 10$ | $^2\Sigma_{3/2^+}(xxx)$, $^2\Sigma_{5/2^+}(xxx)$ | 2056 |
| L=2 N=0 70 $^2 1$ | $\Lambda_{3/2^+}(xxx)$, $\Lambda_{5/2^+}(xxx)$ | 1809 |

The octet partners as nucleon excitation belonging to $^2 8$ in a 70-plet, with spin-parities $N_{3/2^+}$ and $N_{5/2^+}$, should have a mass of 1779 MeV but there are no known states.

## 6.6 Resonances in the third harmonic-oscillator band

The third harmonic-oscillator band is in an intermediate range in which we still observe some states which document the richness of the three-particle dynamics, but the number of seen states is already significantly reduced compared to our expectation. In particular strange baryons are scarce and have mostly no spin-parity determination. In the third band, we expect the following multiplets:

$$(70, 3^-)_{\rm N=0}; (70, 1^-)_{\rm N=1}; (56, 3^-)_{\rm N=0}; (56, 1^-)_{\rm N=1}.$$

We do not necessarily expect two multiplets with the same quantum numbers to lead to a duplication of states. In addition, there is a 70-plet with L=2 and even parity for which we have no evidence. (They require both oscillators to be excited at the same time.) Remember, the oscillator band is $N = 3$ for L=1 and N=1.

$(\mathbf{70}, \mathbf{3}^-)_{\rm N=0}$**:** We find a doublet of nucleon resonances of negative parity, with J=5/2 and 7/2, at 2190 and 2200 MeV, respectively, which match our expectation for the $^2 8(70, 3^-)_{\rm N=0}$ multiplet. The $N_{9/2^-}(2250)$ has intrinsic spin S=3/2 and is assigned to the $^4 8(70, 3^-)_{N=0}$ multiplet. In the $\Lambda$ sector we should expect two $\Lambda_{7/2^-}$ and two $\Lambda_{5/2^-}$, one doublet at 2100 MeV due to the singlet system, one doublet at 2248 MeV belonging to the octet. Only one state is known, the $\Lambda_{7/2^-}(2100)$ which we interpret as singlet state. The $\Sigma_{7/2^-}(2100)$ with its rather low mass weakens somehow the evidence for the assignment of the $\Lambda_{7/2^-}(2100)$ as SU(3) singlet state. The $\Sigma_{7/2^-}(2100)$ is however a 1* resonance only; the $\Lambda_{7/2^-}(2100)$ with its 4* is certainly much more reliable. The $\Sigma(2245)$ - with unknown spin-parity - fits excellently to L=3 spin-1/2 octet hypothesis. One $\Xi$ state, the $\Xi(2370)$, may have L=3 when the mass formula is used as a guide. The $\Delta_{7/2^-}(2200)$ is a natural case to be assigned to $^2 10(70, 3^-)_{N=0}$. Hence we have evidence for all parts of the SU(3) decomposition of the $(70, 3^-)_{N=0}$ super-multiplet, of the $^2 10, ^2 8, ^4 8$ and $^2 1$ multiplets.

$\mathbf{(56, 3^-)_{N=0}}$**:** The $(56, 3^-)_{N=0}$ super-multiplet requires the existence of a $\Delta_{9/2^-}$ resonance at 2223 MeV. There is no evidence for such a state. At L=1, S=3/2 decuplet states are forbidden; we assume that this selection also holds for higher odd-orbital-angular-momentum resonances. Nucleon resonances with spin 1/2 do exist; they can however also be assigned to the $(70, 3^-)_{N=0}$ as discussed above. Hence we do not find evidence that the $(56, 3^-)_{N=0}$ super-multiplet is needed dynamically.

$\mathbf{(56, 1^-)_{N=1}}$**:** Resonances with $(56, 1^-)_{N=1}$ exist; the three states $\Delta_{5/2^-}(1930)$, $\Delta_{3/2^-}(1940)$, and $\Delta_{1/2^-}(1900)$ are likely members of this super-multiplet.

$\mathbf{(70, 1^-)_{N=1}}$**:** We now turn to the $(70, 1^-)_{N=1}$ resonances. $N^*$'s belonging to the $(70, 1^-)_{N=1}$ super-multiplet are missing in the Tables; they should have a mass of 1866 MeV. The SAPHIR Collaboration has suggested two states which would fit to this prediction, a $N_{1/2^-}(1897)$ [28] observed in the $N\eta'$ decay mode and a $N_{3/2^-}(1895)$ [29] decaying into $K^+\Lambda$. $\Delta^*$'s in the $(70, 1^-)_{N=1}$ super-multiplet should have a mass of 1950 MeV. The three states $\Delta_{5/2^-}(1930)$, $\Delta_{3/2^-}(1940)$, and $\Delta_{1/2^-}(1900)$ are naturally assigned to the $(56, 1^-)_{N=1}$ super-multiplet; the two states $\Delta_{3/2^-}(1940)$ and $\Delta_{1/2^-}(1900)$ could also come from the $(70, 1^-)_{N=1}$ super-multiplet; according to the mass formula there is double-occupation, two states are hidden.

No $\Lambda$ resonances are known which could be ascribed to the $(70, 1^-)_{N=1}$ super-multiplet, but there are the two states $\Sigma_{1/2^-}(2000)$ and $\Sigma_{3/2^-}(1940)$ which are predicted to have a mass of 1977 MeV.

In summary, in the third harmonic oscillator band we find evidence for the three super-multiplets $(70, 3^-)_{N=0}$, $(56, 1^-)_{N=1}$ and $(70, 3^-)_{N=1}$, while the $(56, 1^-)_{N=0}$ and the 20-plets are missing.

## 6.7 High-mass resonances

At high masses, essentially only $N^*$'s and $\Delta^*$'s with high total angular momenta are known. For even parity, nucleons are in a 56-plet: there are the $N_{9/2^+}(2220)$ and the $N_{13/2^+}(2700)$ but there are no known $N_{11/2^+}$ or $N_{15/2^+}$ with S=3/2 and L=4 or 6, respectively. There is only one negative-parity nucleon with orbital angular momentum L=5, the $N_{11/2^-}(2600)$ which is likely a $^2 8(70, L^-)_{N=0}$ resonance. We mention that the two states $N_{1/2^-}(2090)$ and $N_{3/2^-}(2080)$ can be interpreted as second radial excitations.

The $\Delta$ resonances with even parity are naturally given S=3/2 - as expected for decuplet states in the 56-plet: the $\Delta_{11/2^+}(2420)$ and $\Delta_{15/2^+}(2950)$ evidence that the intrinsic spin is

S=3/2. The $\Delta_{11/2^+}(2420)$ is accompanied by the $\Delta_{7/2^+}(2390)$ and $\Delta_{9/2^+}(2300)$. We assume they have orbital angular momentum L=4, coupling to J=5/2 (missing), J=7/2, 9/2 and 11/2. Negative-parity $\Delta$ resonances also like to have S=3/2: there are the $\Delta_{9/2^-}(2400)$ and the $\Delta_{13/2^-}(2750)$ which should - based on the Regge trajectories - have intrinsic orbital angular momenta 3 and 5, respectively. In the first and third harmonic oscillator levels, $\Delta$'s without additional radial excitation (N=0) should be in a 70-plet and have spin 1/2. Therefore we assume that the latter two states also have one quantum of radial excitation energy (N=1). The calculated masses support this conjecture. L=3 and S=3/2 couple to 3/2, 5/2, 7/2 and 9/2. The $\Delta_{5/2^-}(2350)$ has the right mass and quantum numbers to fall into this super-multiplet, together with the $\Delta_{9/2^-}(2400)$. The $\Delta_{1/2^-}(2150)$ could be a second radial excitation, like the $N_{1/2^-}(2090)$ and $N_{3/2^-}(2080)$.

In summary, we observe $N^*$ and $\Delta^*$ resonances with even parity as members of a 56-plet. Odd-parity nucleon resonances are observed in $^2 8(70, L^-)_{N=0}$ multiplets, $\Delta$'s in the $^4 10(56, L^-)_{N=1}$ multiplets. The reason for these preferences are unknown; we remind the reader that not all solutions of the Hamiltonian need to be realized in nature as stable rotations. The assignments of high-mass $\Sigma$, $\Xi$ and $\Omega$ states given in the Tables (for which spin-parities are not measured) are based on these selection rules.

## 6.8 Suggestions for further experiments

A first point of experimental interest would be to confirm the reliability of the mass formula. Both, the quark model using one-gluon-exchange [15] and the quark model with instanton-induced interactions [17] predict the three states $\Delta_{5/2^-}(1930)$, $\Delta_{3/2^-}(1940)$, and $\Delta_{1/2^-}(1900)$ at masses in the 2200 MeV region. It has been proposed [30] to search for the $\Delta_{3/2^-}(1940)$ in its hypothetical $\Delta_{3/2^-}(1940) \to \Delta_{3/2^+}(1232)\eta$ decay. A confirmation would validate the model proposed here and would be difficult to incorporate in those two quark models.

The $(70, 1^-)_{N=1}$ multiplet predicts nucleon resonances with spin 1/2 at 1866 MeV. The SAPHIR collaboration suggested a $N_{1/2^-}$ resonance at 1897 MeV [28]. The resonance was observed below the $N\eta'$ threshold in the $N\eta'$ decay mode. A further resonance is claimed, a $N_{3/2^-}$, at 1895 MeV in its decay into $K^+\Sigma^0$ [29]. The two resonances are perfectly compatible with forming part of the $(70, 1^-)_{N=1}$ multiplet. A high-statistics study including polarization observables should decide if these claims are justified.

The model predicts a further positive-parity $N^*$ doublet with J=5/2 and 3/2 between the low mass states at 1700 MeV and the states at 1950 MeV. These states should be observable at Jlab and at ELSA in two-pion photo- or electroproduction experiments [21, 22]. The 1900 to 2000 MeV region hosts a large number of $N^*$ and $\Delta^*$ states and further resonances are expected to hide in this mass range. A very detailed study of several final states is needed to test this prediction.

It would be highly interesting to explore the high-mass range to see if the selection rules are artifacts of the analyses which identify most easily the highest partial wave, or if these selection rules really show how a high-spin baryon adjusts its shape and internal degrees of freedom to the large centrifugal forces.

# 7 Interpretation

## 7.1 A new interpretation of strong QCD

Given the simplicity of the mass formula, the agreement between experimental values and the prediction is remarkable. Hence we have to discuss what the reason for this good agreement might be. At the first glance there is a clear contradiction. All baryon resonances are rather well reproduced with a model which takes the string constant and the radial excitation energy from mesons, from a quark-antiquark system. Obviously baryon resonances behave like quark-diquark excitations.

Diquark models of baryons have been suggested frequently but they have an intrinsic weakness. Let us consider the lowest orbital angular momentum excitation, the $N_{3/2^-}(1520)D_{13}$ resonance. The intrinsic orbital angular momentum is 1, it combines with the intrinsic spin 1/2 to a total angular momentum J=3/2. Now, not only one quark is excited to an orbital angular momentum 1, both harmonic oscillators are coherently excited. This splitting of the oscillatory motion is required by Fermi-Dirac statistics, by the requirement that the wave function should be antisymmetric with respect to the exchange of two quarks. Diquark models usually circumvent this demand by assuming that diquarks are tightly bound so that anti-symmetrization of quark pairs, one outside and one inside of the diquark, is not required. Experimentally there is no support for a tightly bound diquark; a diquark model is also not consistent with the forces expected from QCD.

There is one way out: We may assume that baryon resonances are quark-diquark excitations in color space. So we assume that the dynamics is given by color. Antisymmetrization can be arranged by flavor exchange. So a red up-quark may turn into a red down-quark. It maintains color and exchanges its flavor. Now we have to explain why color exchange is a slow process.

We propose a somehow unusual concept for constituent quarks. We assume that the strong color-forces polarize the quark and gluon condensates of the QCD vacuum. The current quark plus its polarization cloud forms what we call now a constituent quark of defined color. Color exchange is screened by the polarization cloud. When a gluon is emitted it is re-absorbed in the polarization cloud. Color propagates only stochastically from one color center to another center within a polarization cluster, globally the constituent quark keeps its color for a finite time, which is longer than the life time for flavor exchange and comparable to the life time of the baryon resonance. Flavor is not a property of a constituent quark. The matrix element governing color exchange is not known; we estimate it to be in the order of $\Lambda_{\rm QCD}$.

In contrast to color exchange there is a fast flavor exchange. Flavor exchange is not shielded by the polarised condensates; flavor propagates freely in the QCD vacuum. Flavor exchange is possible via long-range meson-exchange or by instanton interactions at the surface of two neighboring colored constituent quarks. Flavor exchange acts at a time scale given by the constant responsible for chiral symmetry breaking, $\Lambda_\chi$. In this picture confinement originates from Pomeron-exchange-like forces transmitted by the polarization of the vacuum condensates. Different SU(6) multiplets with the same quantum numbers differ in their internal spin-flavor structure which has no significance for the color wave function, and thus no importance for the mass.

Color is usually not considered to be an observable quantity. For the baryon ground states, this assumption is true. All three quarks are in the same phase-space cell, the quarks cannot be localized and their individual properties are not observables. This argument is not applicable to excited baryons. If a baryon has an intrinsic orbital angular momentum of, let us say, L=4, one quark is well separated in phase space from the two other quarks. Its color can be defined and can be shielded by its self-generated polarization cloud.

This picture suggests that the largest contribution to the mass of a hadron comes from the mass density of the polarization cloud and the hadronic volume. This idea can be tested in a string model of quark-diquark interactions. We assume that the polarization cloud between quark and diquark is concentrated in a rotating flux tube or a rotating string with a homogeneous mass density. The length of the flux tube is $2r_0$, its transverse radius R. The velocity at the ends may be the velocity of light. Then the total mass of the string is given by [12]

$$\mathrm{Mc}^2 = 2\int_0^{r_0} \frac{\mathrm{kdr}}{\sqrt{1-\mathrm{v}^2/\mathrm{c}^2}} = \mathrm{kr}_0\pi \tag{10}$$

and the angular momentum by

$$\mathrm{L} = \frac{2}{\hbar \mathrm{c}^2}\int_0^{r_0} \frac{\mathrm{krvdr}}{\sqrt{1-\mathrm{v}^2/\mathrm{c}^2}} = \frac{\mathrm{kr}_0^2\pi}{2\hbar \mathrm{c}} \tag{11}$$

The orbital angular momentum is proportional to

$$\mathrm{L} = \frac{1}{2\pi \mathrm{k}\hbar \mathrm{c}}\mathrm{M}^2 \tag{12}$$

From the slope in Fig. 4 we find $k = 0.2\,\mathrm{GeV}^2$ and a radius of

$$\mathrm{r}_0\left(\Delta_{15/2^+}(2950)\right) = 4\mathrm{fm} \tag{13}$$

According to the Nambu model, excited quark and the diquark in the $\Delta_{15/2^+}(2950)$ are separated by 8 fm!

The volume of the flux tube is $2\pi \mathrm{R}^2\mathrm{r}_0$, the mass density

$$\rho = \frac{k}{2R^2c^2}. \tag{14}$$

We now assume that the mass density in the $\Delta(1232)$ is the same as the one in the flux tube. We thus relate

$$\frac{4}{3}\pi \mathrm{R}^3 \cdot \rho = \mathrm{M}_{\Delta(1232)} \tag{15}$$

which gives a radius of the polarization cloud of the $\Delta(1232)$ of 0.6 fm (and 0.37fm for the $\rho$). This is not unreasonable, even though smaller than the RMS charge radius of the proton. However, an additional pion cloud may increase the charge radius.

## 7.2 Consequences of the colored-constituent-quark concept

The assumption that constituent quarks have a defined color, and that color exchange is shielded by the polarization cloud offers a new interpretation for a large number of phenomena which are not yet understood.

**Confinement:** When two quarks are separated, the volume in which the QCD vacuum is polarized increases with the quark-quark separation. The net color charge does not change, hence the energy stored in the polarized condensates increases linearly: the confinement potential is a linear function of the quark separation. This interpretation of the confinement potential follows immediately from the assumption that color exchange between two quarks is a slow process and this in turn is the consequence of the similarity of the meson and of the baryon string tension.

**Structure functions:** The polarization clouds surrounding the current quarks are of course seen in deep inelastic scattering, the quarks directly, the gluons through their contribution to the total momentum.

**The spin crisis:** It was a surprise when it was discovered that the proton spin is not carried by quarks. The success of the naive quark model in the prediction of the ratios of magnetic moments of octet baryons seemed to be a solid basis for the assumption that the spin of the proton should be carried by its 3 valence quarks. But this naive expectation fails, the contribution of all quark- and antiquark-spins to the proton spin is rather small [13]. A large fraction from the proton spin must be carried by the intrinsic orbital angular momenta of quarks or by orbital or spin contributions of gluons. We assume that the magnetic moment of the spin induces polarization into the condensates. The polarized gluon condensates provide a gluonic contribution to the proton spin, the quark condensate a spin and orbital contribution to it. Orbital angular momenta of quarks enter because the quarks in the condensate are pairwise in the $^3\mathrm{P}_0$ state. The orientation defined by direction of the current-quark spin may induce internal currents which contribute to the magnetic moment.

An analogy can be found in superconductivity. If a magnet moment is implanted into a superconducting material, the superconductivity may be destroyed. If it is maintained the Cooper pairs will be polarized and the currents adjust to take over part of the magnetic moment of the alien element.

**The $^3\mathrm{P}_0$ model:** A further example for the usefulness of the concept proposed here is the $^3\mathrm{P}_0$ model for meson and baryon decays. According to this model the quantum numbers of a $\mathrm{q\bar{q}}$ pair, created in a decay process, have the quantum numbers of the vacuum. These quantum numbers are preserved, when a $\mathrm{q\bar{q}}$ pair from the condensate is shifted to the mass shell.

**Hybrids and glueballs:** Finally, the picture also provides a new interpretation of glueballs and hybrids. In the flux tube model hybrids are thought of as quark-antiquark pairs connected via a gluonic string forming a flux tube. This flux tube can be exited and such excitations are called hybrids if they are resonant. Here the question arises if the polarization status of the gluon and quark condensates can undergo oscillatory motions. This is a dynamical question and not only a question of mean forces and of an energy minimum. There is, e.g., the possibility that the polarization cloud supports longitudinal oscillations, sound-like waves. These can be identified with radial excitations. Their existence is well established; this does not imply that the polarization cloud also supports transverse oscillations. We emphasize that in the baryon spectrum, there is no evidence for additional states, neither hybrid states (with the flux tube excited) nor for penta-quarks.

Glueballs can be seen as polarization clouds without a color charge driving the polarization. Here, the question is if the gluon and quark condensates support soliton-like solutions which propagate in free space and which manifest themselves as energy bumps. A recent critical review on the status of hybrids and of the scalar glueball can be found in [31].

**The Color-Constituent-Quark model and lattice QCD** The problems which are discussed here seem unsolvable within perturbative QCD or in quenched lattice calculations. The effect a color charge induces in the quark and gluon condensates are not within the range of present-day lattice QCD. Even the inclusion of a small number of virtual $\mathrm{q\bar{q}}$ pairs is unlikely to represent the full complexity of the QCD vacuum and its response to a color source.

# 8 Summary

We have proposed a new baryon mass formula which reproduces with good precision most of the available baryon masses. The formula is based on the observation that the slope of mesonic and baryonic Regge trajectories are the same, that radial excitations of mesons and baryons have similar level spacings and that mass splittings ascribed to one-gluon exchange like color-magnetic spin-spin and spin-orbit interactions do not play an important role for baryon masses. The octet-decuplet mass splitting is shown to depend on the symmetry of the wave function; the splitting is not induced by spin-spin color-magnetic interactions but by instantons. The richness of the baryon spectrum prevents an interpretation of baryon resonances as quark-diquark excitations. We propose therefore that the masses are given by the dynamics of colored quark clusters, constituent quarks of defined color. Fast flavor exchange fulfills the requirement of Fermi-Dirac symmetry.

In our view, the three quarks of a baryon polarize the vacuum condensates which then shield the color charge preventing a fast color exchange. As color exchange is slow, there is no color-magnetic interaction. Baryon masses depend on the dynamics of colored constituent quarks or colored quark clusters. Flavor is not a property of constituent quarks.

This view of constituent quarks as colored quark clusters has wide-reaching consequences which are briefly outlined. The view offers a language in which confinement can be described and the proton spin crisis be qualitatively understood. It offers new interpretations for so-called gluonic excitation modes in hadron spectroscopy, of hybrids and glueballs. We are convinced that baryons provide key issues for an improved understanding of QCD in the confinement region.


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