Diquark correlations in baryon spectroscopy and holographic QCD

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We introduce an improved mass formula for the nucleon and delta resonances and show how it emerges from AdS/QCD in a straightforward extension of the "metric soft wall" gravity dual. The resulting spectrum depends on just one adjustable parameter, characterizing confinement-induced IR deformations of the anti-de Sitter (AdS) metric, and on the fraction of "good" (i.e. maximally attractive) diquarks in the baryon's quark model wave function. Despite its simplicity, the predicted spectrum describes the masses of all 48 observed light-quark baryon states and their linear trajectory structure with unprecedented accuracy.

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The arguably most prominent and pervasive structure in the known hadron spectrum consists of (approximately) linear Regge-type trajectories of equal slopes on which the square masses M^2 of excited states organize themselves in Chew-Frautschi plots, i.e. as a function of either the angular momentum L (or total spin J) or the radial excitation level N. The QCD-based understanding of these trajectories and their relation to linear quark confinement remains one of the pre-eminent challenges of strong-interaction physics.

In this Letter we propose a mass formula for the trajectories in the light-quark baryon (i.e. nucleon and delta) sector which improves upon the recent AdS/QCD [1] "metric soft-wall" (ms) prediction [2]

$$M_{N,L}^{(\text{ms})2} = 4\lambda^2 \left(N + L + \frac{3}{2}\right) \quad (1)$$

and show how it emerges from a straightforward extension of the ms gravity dual. More specifically, while Eq. (1) works very well in the Δ sector (i.e. all observed Δ^* resonance states lie within errors on the predicted trajectory [3] whose empirical slope is used to determine the AdS_5 deformation parameter $\lambda = 0.52$ GeV), we will show that its predictions for the nucleon excitation spectrum can be substantially improved by adding the correction

$$\Delta M_{\kappa_{\text{gd}}}^2 = -2\left(M_\Delta^2 - M_N^2\right)\kappa_{\text{gd}} \quad (2)$$

which solely depends on the resonances' diquark content. The latter enters through the good diquark fraction κ_{gd} which we define as the fraction of the space-spin-flavor baryon wavefunction in the $\text{SU}(2)_s \otimes \text{SU}(3)_f$ product representation which contains the most attractive, i.e. the "good" (0^+ color-antitriplet iso-singlet) diquark. This implies in particular $\kappa_{\text{gd}} = 0$ for all Δ and spin-3/2 nucleon resonances, $\kappa_{\text{gd}} = 1/4$ for spin-1/2 nucleons in the $70_{\text{SU}(6)}$ representation and $\kappa_{\text{gd}} = 1/2$ for nucleons in the $56_{\text{SU}(6)}$ multiplet. The mass correction (2) renders the long-suggested [4] and recently reemphasized [5, 6] importance of attractive diquark correlations in baryon structure explicit. Interest in such correlations currently extends to phenomena ranging from exotic hadrons [6] to color-superconducting phases of ultradense matter [7].

In order to compare Eqs. (1), (2) to experimental data, one needs to assign intrinsic orbital and spin angular momenta L and S to the observed states. At first glance this seems unfeasible since only total spin J and parity P are measured. However, the data show regularities which bring additional order into the spectrum (see Table I). The three-body dynamics of the quark model implies that the number of expected states grows dramatically with increasing excitation energy. We will discuss these states in the non-relativistic harmonic oscillator (h.o.) approximation. Although the h.o. states may mix, we will show that the leading L and S configurations can nevertheless be determined. In terms of the Jacobi coordinates ρ and λ, the two h.os. exhibit orbital (l_ρ, l_λ) and radial (n_ρ, n_λ) excitations with $L{=}l_\rho{+}l_\lambda$ and $N{=}n_\rho{+}n_\lambda$, and the energy levels develop a shell structure which manifests itself experimentally in a series of resonance regions.

The Particle Data Group lists 22 nucleon and 22 Δ resonances with known J and P [8]. We use all of them except for the one-star $\Delta(1750)$ – which is special and requires a devoted discussion – and except for the two-star $\Delta(2000)$ with $J^P{=}5/2^+$ for which three mass values are quoted, 1724, 1752, and 2200 MeV. The long debated question whether the Roper resonance $N(1440)$ is the first radial excitation of the nucleon has recently found an affirmative answer in electro-production experiments [9]. $\Delta(1600)$ with $J^P{=}3/2^+$ is the analogue state in the Δ spectrum; we asign $L, N{=}0{,}1$ to both. We further conjecture (**A**) that radial and orbital excitations cost the same energy, in line with the $N + L$ dependence of Eq. (1).

The two states $N(1535)$ and $N(1520)$ with $J^P{=}1/2^-$, $3/2^-$, and the three states $N(1650), N(1700), N(1675)$ with $J^P{=}1/2^-, 3/2^-, 5/2^-$ naturally form a spin doublet with $L{=}1,S{=}1/2$ and a $L{=}1,S{=}3/2$ triplet. Symmetry arguments forbid a symmetric spin wave function for the lowest-mass Δ states with $L{=}1$, and indeed only the $L{=}1, S{=}1/2$ spin doublet $\Delta(1620)$, $\Delta(1700)$ is ob-

TABLE I: Masses and quantum numbers of N and Δ states, together with the predictions of Eqs. (1) and (2).

L,N	$\kappa_{\rm gd}$	Resonance					Pred.
0,0	$\frac{1}{2}$	$N(940)$				input:	**0.94**
0,0	0	$\Delta(1232)$					1.27
0,1	$\frac{1}{2}$	$N(1440)$					1.40
1,0	$\frac{1}{4}$	$N(1535)$	$N(1520)$				1.53
1,0	0	$N(1650)$	$N(1700)$	$N(1675)$			1.64
1,0	0	$\Delta(1620)$	$\Delta(1700)$		L,N=0,1:	$\Delta(1600)$	1.64
2,0	$\frac{1}{2}$	$N(1720)$	$N(1680)$		L,N=0,2:	$N(1710)$	1.72
2,0	0	$N(1900)$	$N(1990)$	$N(2000)$	$\Delta(1910)$	$\Delta(1920)$	1.92
2,0	0	$\Delta(1905)$	$\Delta(1950)$	$\Delta(1900)^*$	$\Delta(1940)^*$	$\Delta(1930)^*$	1.92
0,3	$\frac{1}{2}$	$N(2100)$					2.03
3,0	$\frac{1}{4}$	$N(2070)$	$N(2190)$	L,N=1,2:	$N(2080)$	$N(2090)$	2.12
3,0	0	$N(2200)$	$N(2250)$	$\Delta(2223)$	$\Delta(2200)$	L,N=1,2: $\Delta(2150)$	2.20
4,0	$\frac{1}{2}$	$N(2220)$					2.27
4,0	0	$\Delta(2390)$	$\Delta(2300)$	$\Delta(2420)$	L,N=3,1:	$\Delta(2400)$ $\Delta(2350)$	2.43
5,0	$\frac{1}{4}$	$N(2600)$					2.57
6,0	$\frac{1}{2}$	$N(2700)$					2.71
6,0	0	$\Delta(2950)$			L,N=5,1:	$\Delta(2750)$	2.84

*: L,N=1,1.

served. L=1, S=3/2 would form a triplet; symmetry requires excitation of the radial wave function to N=1. These states are seen as $\Delta(1900), \Delta(1940)$, and $\Delta(1930)$ having J^P=$1/2^-, 3/2^-, 5/2^-$ quantum numbers. Their low mass – compared to quark model calculations – arises from conjecture (**A**). Furthermore, we conjecture (**B**) that all negative parity Δ excitations with S=3/2 have N=1.

In addition, there are three positive-parity nucleon resonances $N(1900), N(1990), N(2000)$ with quantum numbers $J^P = 3/2^+, 5/2^+, 7/2^+$, four Δ states $\Delta(1910)$, $\Delta(1920), \Delta(1905), \Delta(1950)$ with $J^P = 1/2^+, 3/2^+, 5/2^+$, $7/2^+$, and no other close-by state with these quantum numbers. It is therefore natural to assign L=2 and spin S=3/2, yielding a Δ and a nucleon quartet; the latter quartet is completed by the recently proposed $N(1880)$ with $J^P = 1/2^+$ [14].

The $J^P = 1/2^+$ $N(2100)$ has no spin-parity partner; we interpret it as the third excitation in the series $N(940)$, $N(1440), N(1710), N(2100)$. The two states $N(2070)$, $N(2190)$ could form a L=3, S=1/2 spin doublet, and the three states $N(2200), N(2190), N(2250)$ an incomplete quartet where the lowest angular momentum state with $J^P = 3/2^-$ is missing [10]. The $N(2190)$ assignment is ambiguous. The $J^P = 1/2^-$ $N(2080)$ must have L=1 and could be the second radial excitation of $N(1535)$. Its spin doublet partner is then $N(2090)$ as the second $N(1520)$ excitation, its isospin partner is $\Delta(2150)$. The isospin partner of $N(2190)$ is missing. First excitations have been suggested: $N(1905)$ with $J^P = 1/2^-$ [11, 12] and $N(1960)$ with $J^P = 3/2^-$ [12, 13, 14]. Their predicted mass is 1.82 GeV.

Due to its low mass, the $J^P = 9/2^-$ state $N(2250)$ cannot have $L = 5$ (apart from a small admixture), rather it must have $L = 3, S = 3/2$. The three other members with lower J are missing. $\Delta(2223)$ (suggested in [15]) and $\Delta(2200)$ with $J^P = 5/2^-, 7/2^-$ are natural candidates for the L=3, S=1/2 doublet. At a bit higher mass, $\Delta(2400)$ with $J^P = 9/2^-$ is observed. According to our conjecture (**B**), it has L=3, S=3/2, N=1. $\Delta(2350)$ with $J^P = 5/2^-$ could be its spin-multiplet partner while $J^P = 3/2^-, 7/2^-$ are missing. Conjecture (**B**) also entails the $L = 5, S = 3/2$, $N = 1$ assignment for the high-mass $J^P = 13/2^-$ resonance $\Delta(2750)$. At 2.4 GeV, a nearly complete spin multiplet with $L = 4, S = 3/2$ is known: $\Delta(2390)(7/2^+)$, $\Delta(2300)(9/2^+)$, $\Delta(2420)(11/2^+)$. The assignments of $N(2700)$ and $\Delta(2950)$, finally, are unambiguous.

Table I reveals the extent of the agreement between the 48 measured mass values and Eqs. (1), (2). In fact, the latter considerably improve upon any dynamical quark model prediction of the full mass spectrum. This supports the validity of both Eqs. (1), (2) and of our quantum number assignments, and it raises the hope that not only Eq. (1) but also the universal correction term (2) may have a transparent origin in holographic QCD. In the following we will show that this is indeed the case. Since the metric soft wall [2] is so far the only AdS/QCD dual which predicts linear square-mass trajectories for baryons (cf. Eq. (1)), we take this background as our starting point. The IR deformed AdS_5 line element

$$ds^2 = g_{MN} dx^M dx^N = a^2(z) \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right) \quad (3)$$

(where η is the four-dimensional Minkowski metric and z the conformal coordinate of the fifth dimension) of the metric soft wall in the baryon sector is characterized by the warp factor [2]

$$a^{(\rm ms)}(z) = \frac{R}{z} \left(1 + \frac{\lambda^2 z^2}{m_5^{(0)} R} \right) \quad (4)$$

(where R is the curvature radius, $m_5^{(0)}$ the Dirac bulk mode mass and λ the scale which governs the IR deformation). In this geometry the Dirac equation for the string modes $\Psi(x,z)$ dual to nucleons reads

$$\left[i e_A^M \Gamma^A \left(\partial_M + 2 a^{(\rm ms)-1} \partial_M a^{(\rm ms)} \right) - m_5^{(0)} \right] \Psi(x,z) = 0 \quad (5)$$

(where $e_M^A = a \delta_M^A$ are fünfbeins and Γ^A Dirac matrices). By iteration, Eq. (5) can be cast into the Sturm-Liouville form

$$\left[-\partial_z^2 + V_\pm^{(\rm ms)}(z) \right] \psi_\pm^{(\rm ms)}(z) = M^{(\rm ms)2} \psi_\pm^{(\rm ms)}(z) \quad (6)$$

where the modes $\psi_\pm(z) = \left(\lambda z e^{-A} \right)^{-2} \varphi_\pm(z)$ describe the rescaled chiral components of the bulk spinor

$$\Psi(x,z) = \int \frac{d^4 k}{(2\pi)^4} e^{-ikx} \times \times \left[\varphi_+^{(k)}(z) P_+ + \varphi_-^{(k)}(z) P_- \right] \hat{\Psi}^{(4)}(k) \quad (7)$$

($P_\pm \equiv \left(1 \pm \gamma^5\right)/2$) and the boundary spinor $\hat{\Psi}^{(4)}$ solves the $d=4$ Dirac equation $\left(\gamma^a k_a - |k|\right) \hat{\Psi}^{(4)}(k) = 0$. The metric soft-wall potential is [2]

$$V^{(\mathrm{ms})}_{m_5^{(0)},\pm}(z) = \pm\partial_z \left(a^{(\mathrm{ms})} m_5^{(0)}\right) + a^{(\mathrm{ms})2} m_5^{(0)2} \quad (8)$$

and the corresponding eigenfunctions $\psi_\pm^{(\mathrm{ms})}$ can be found analytically [2]. After implementing the boundary behavior $\varphi_+(z) \overset{z\to 0}{\longrightarrow} z^{\bar{\tau}}$ required by the AdS/CFT dictionary for baryon interpolators of twist dimension $\bar{\tau}$, which amounts to replacing $m_5^{(0)} R \ \to \bar{\tau} - 2 = L + 1$ [16], the eigenvalue spectrum (1) emerges.

As mentioned above, this spectrum agrees very well with data in the Δ sector but can be substantially improved in the nucleon sector by adding the correction term (2) which depends on the diquark content of the baryon resonances. At first it might seem impossible to gain holographic access to this diquark information because diquarks and their operators are gauge dependent whereas only gauge-invariant operators have well-defined duals. However, diquark information may enter indirectly through gauge-invariant baryon interpolating fields whose general form [17]

$$\eta_t(x) = 2\left[\eta_{\mathrm{pd}}(x) + t\eta_{\mathrm{sd}}(x)\right] \quad (9)$$

at leading twist (i.e. with the minimal scaling dimension 9/2) contains a pseudoscalar diquark in $\eta_{\mathrm{pd}} = \varepsilon_{abc}\left(u_a^T C d_b\right)\gamma^5 u_c$ and a "good" scalar diquark in $\eta_{\mathrm{sd}} = \varepsilon_{abc}\left(u_a^T C\gamma^5 d_b\right) u_c$. (Higher-twist interpolators contain additional covariant derivatives.)

The interpolators (9) are expected to have enhanced overlap with nucleon states of the corresponding diquark content and can thus be related to the κ_{gd} of these states [23]. Since $t \to \infty$ corresponds to maximal "good" diquark content and $t = -1$ (the "Ioffe current") to an exclusively "bad" diquark content, a simple approximation to this relation may e.g. be

$$t(\kappa_{\mathrm{gd}}) = \frac{1}{2\kappa_{\mathrm{gd}} - 1}. \quad (10)$$

The different diquark content of the baryon interpolators η_t can affect properties of their dual modes in several ways. For once, the AdS/CFT dictionary relates the mass m_5 of a dual string mode to the scaling or twist dimension of the corresponding interpolator. While all equal-twist baryon interpolators have the same classical twist dimension $\bar{\tau} = L + 3$ and hence correspond to dual modes of equal mass $m_5^{(0)} = (L+1)/R$ [16], their anomalous dimensions will generally depend on t and therefore induce κ_{gd} dependent mass corrections $\Delta m_5(\kappa_{\mathrm{gd}})$. Such corrections could also result from κ_{gd} dependent couplings of the Dirac modes to condensate-related spin-0 bulk fields. Since at present no reliable QCD information on nonperturbative anomalous dimensions exist, we will treat Δm_5 as an adjustable parameter to be determined in bottom-up fashion from the observed baryon spectrum (as similarly suggested in Ref. [21]).

We are now going to show that such a bulk mass correction $\Delta m_5(\kappa_{\mathrm{gd}})$ indeed results in the desired spectral correction (2) and minimally just requires a small additional IR deformation of the soft wall metric (3). To this end, we first observe that a shifted dual mode mass

$$m_5 = m_5^{(0)} + \Delta m_5 = \frac{L + \Delta m_5 R + 1}{R} \quad (11)$$

with the corresponding IR deformation

$$a(z) = \frac{R}{z}\left(1 + \frac{\lambda^2 z^2}{L + \Delta m_5 R + 1}\right) \quad (12)$$

of the warp factor (4) turns the Sturm-Liouville potential (8) into

$$V_{L,\pm}(z) = V^{(\mathrm{ms})}_{m_5^{(0)}+\Delta m_5,\pm}(z) = V^{(\mathrm{ms})}_{L+\Delta m_5 R,\pm}(z). \quad (13)$$

Hence the eigenfunctions $\psi_\pm$ in this potential can be obtained from the analytical solutions $\psi^{(\mathrm{ms})}_{N,L,\pm}$ of Ref. [2] as $\psi_{N,L,\kappa_{\mathrm{gd}},\pm}(z) = \psi^{(\mathrm{ms})}_{N,L+\Delta m_5 R,\pm}(z)$ (which are well-defined for $\Delta m_5 R > -3/2$). Finally, after specifying

$$\Delta m_5(\kappa_{\mathrm{gd}}) = \frac{\Delta M^2_{\kappa_{\mathrm{gd}}}}{4\lambda^2 R} \quad (14)$$

(so that $\Delta m_5 R \gtrsim -0.7$ and $m_5 > 0$) the eigenvalue spectrum (1) indeed turns into

$$M^2_{N,L} = M^{(\mathrm{ms})2}_{N,L+\Delta m_5 R} = 4\lambda^2\left(N + L + \frac{3}{2}\right) + \Delta M^2_{\kappa_{\mathrm{gd}}}. \quad (15)$$

The above derivation also sheds new light on the physics behind this spectrum. Indeed, it shows that the dual modes $\psi_{N,L,\kappa_{\mathrm{gd}},\pm}$ corresponding to larger κ_{gd} (with identical N, L) begin to feel the soft wall at smaller z (cf. Eq. (12)) and therefore extend less into the fifth dimension. This reflects the additional attraction and translates into a smaller size of baryon states with larger κ_{gd}.

A more elaborate alternative for generating the spectrum (15) would take the RG flow of the anomalous dimensions (or additional bulk fields) into account [24]. According to the gauge/gravity correspondence, this renormalization-scale dependence translates into a z dependent $\Delta m_5(z)$. Since the above adaptation of the eigenfunctions then ceases to work, one may instead attempt to generate the mass correction directly in the mode potential,

$$V_{L,\pm}(z) = V^{(\mathrm{ms})}_{L,\pm}(z) + \Delta M^2_{\kappa_{\mathrm{gd}}}, \quad (16)$$

while both the warp factor (4) and the eigenfunctions of the metric soft wall remain unchanged. Since the z

dependent mass correction $\Delta m_{5,\pm}(z) \equiv \tilde{m}_{\pm}(z)$ results in the modification

$$\Delta V_{\pm}(z) = \pm\left(a^{(\mathrm{ms})}\tilde{m}_{\pm}\right)' + a^{(\mathrm{ms})2}\tilde{m}_{\pm}\left(2m_5^{(0)} + \tilde{m}_{\pm}\right) \tag{17}$$

(the prime denotes ∂_z) of the soft-wall mode potential (8), a z (and L) independent shift $\Delta V_{\pm} = \Delta M^2_{\kappa_{\mathrm{gd}}}$ is obtained when the $\tilde{m}_{\pm}(z)$ solve the nonlinear differential equations

$$\begin{aligned} 0 \;=\; & \pm\tilde{m}'_{\pm} + \left(2\frac{L+1}{R}a^{(\mathrm{ms})} \pm \frac{a^{(\mathrm{ms})\prime}}{a^{(\mathrm{ms})}}\right)\tilde{m}_{\pm} \\ & + a^{(\mathrm{ms})}\tilde{m}_{\pm}^2 - \frac{\Delta M^2(\kappa_{\mathrm{gd}})}{a^{(\mathrm{ms})}}. \end{aligned} \tag{18}$$

General solutions of these equations indeed exist and, subject to the boundary condition $\tilde{m}_{\pm}(z) \stackrel{z\to 0}{\longrightarrow} 0$, read

$$\tilde{m}_{\pm}(z) = \frac{4(L+1)\Delta m_5 \lambda^2 z^2}{L+1+\lambda^2 z^2} f_{\pm}(z) \tag{19}$$

with

$$f_{+}(z) \;=\; \frac{1}{2L+3}\frac{{}_1F_1\left(1-\Delta m_5 R, L+\frac{5}{2}, -\lambda^2 z^2\right)}{{}_1F_1\left(-\Delta m_5 R, L+\frac{3}{2}, -\lambda^2 z^2\right)}, \tag{20}$$

$$f_{-}(z) \;=\; \frac{1}{2L+1}\frac{{}_1F_1\left(1+\Delta m_5 R, -L+\frac{1}{2}, \lambda^2 z^2\right)}{{}_1F_1\left(\Delta m_5 R, -L-\frac{1}{2}, \lambda^2 z^2\right)}, \tag{21}$$

where the constant Δm_5 is given by Eq. (14) and ${}_1F_1(a,b,x)$ are Kummer functions [22]. Inserted into Eq. (17), the solutions (19) reproduce Eq. (16) and hence the spectrum (15). However, it is not immediately obvious how the chirality dependence of the mass functions (19) can arise directly from a modified Dirac mass term.

To summarize, we have presented a light-quark baryon mass formula with only one adjustable parameter (related to the string tension of linear quark confinement) which reproduces the masses of all 48 observed nucleon and Δ resonances with far better accuracy than e.g. quark model predictions (although the latter depend on a substantially larger number of parameters). In addition, our spectrum relates the trajectory slopes to the Δ ground-state mass and reveals a strikingly systematic role of the good diquark fraction κ_{gd} in baryon spectroscopy.

We have furthermore shown how this spectrum emerges in holographic QCD, namely by means of string-mode mass corrections in the metric soft wall background which can be naturally traced to the varying diquark content of the QCD baryon interpolators. The latter reflects itself e.g. in different anomalous dimensions and potentially in diquark-content dependent couplings of the dual modes to condensate-related spin-0 bulk fields. (It would be interesting to gain independent quantitative insight into these effects, e.g. by lattice calculations of the QCD baryon interpolators' anomalous dimensions in the infrared.) Finally, our results indicate that baryon sizes decrease with increasing good-diquark content.

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[23] The analoguous dependence of nucleon correlators on interpolators with different diquark content (and their different couplings to instantons) was exploited in QCD sum-rule analyses including small-scale instanton effects [18], in instanton vacuum models [19] and on the lattice [20].

[24] We shall tentatively assume that the RG mixing among anomalous dimensions remains small in the restricted μ (resp. z) range of interest ($z \lesssim \lambda^{-1}$) and does not modify the qualitative results.