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# $SU(3)$ Classification of $p$-Wave $\eta\pi$ and $\eta'\pi$ Systems

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## Abstract

An exotic meson, the $\pi_1(1400)$ with $J^{PC} = 1^{-+}$, has been seen to decay into a $p$-wave $\eta\pi$ system. If this decay conserves flavor $SU(3)$, then it can be shown that this exotic meson must be a four-quark state $(q\bar{q} + q\bar{q})$ belonging to a flavor $\mathbf{10} \oplus \overline{\mathbf{10}}$ representation of $SU(3)$. In contrast, the $\pi_1(1600)$ with a substantial decay mode into $\eta'\pi$ is likely to be a member of a flavor octet.

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An exotic meson, the $\pi_1(1400)$ with $I^G(J^{PC}) = 1^-(1^{-+})$, has been seen to decay into a $p$-wave $\eta\pi$ system [1], [2]. So far this is the only decay mode known for the exotic meson. The purpose of this letter is to show that a $p$-wave $\eta\pi$ system belongs to a flavor $\mathbf{10} \oplus \overline{\mathbf{10}}$ representation by the requirement of Bose symmetrization, if the $\eta$ meson is assumed to be a pure member of the pion octet. This implies that, in the limit of flavor $SU(3)$ conservation in its decay, $\pi_1(1400)$ cannot be a gluonic hybrid ($q\bar{q} + g$) but instead it must belong to a family of four-quark states ($q\bar{q} + q\bar{q}$). In this letter we give, for the first time, a complete set of normalized wave functions corresponding to the family of $\mathbf{10} \oplus \overline{\mathbf{10}}$ coupling to two members of the ground-state ${}^1S_0$. We then explore the consequences of this assignment and suggest experiments which could support or negate this conjecture.

We start the discussion by recalling that a second exotic meson, the $\pi_1(1600)$ with $I^G(J^{PC}) = 1^-(1^{-+})$, is reported to have a substantial decay mode into $\eta'\pi$ [3], [4], [5]. The branching ratios are [4]

$$B[b_1(1235)\pi] \ : \ B[\eta'(958)\pi] \ : \ B[\rho(770)\pi] = 1 \ : \ 1.0 \pm 0.3 \ : \ 1.5 \pm 0.5 \tag{1}$$

A search for a $\eta\pi$ decay mode of the $\pi_1(1600)$ has not been successful.

The first immediate question is: why does the $\pi_1(1600)$ not decay into $\eta\pi$ even though it decays into $\eta'\pi$ and even though the phase space for the former decay mode is much larger. Sometimes it is argued that the strong coupling to $\eta'\pi$ is due to the "gluish" nature of the $\eta'$ and points to a hybrid interpretation of the $\pi_1(1600)$. We do not share this view. Instead, we show that a meson belonging to a flavor octet cannot decay into $\eta\pi$.

First we assume that the $\eta$ meson is the pure isoscalar member of the ground-state ${}^1S_0$ flavor-$SU(3)$ octet, i.e. the other members are the isovector $\pi$ and the isodoublets $K$ and $\bar{K}$. In this scheme then, one has assumed that the $\eta'$ is a pure $SU(3)$ singlet. The exotic quantum numbers enforce the $\eta\pi$ system to be in a relative $p$ wave; hence the $\eta\pi$ system must belong to the antisymmetric octet $\mathbf{8_2}$. This is, however, not possible. To show this, one needs to examine its nonstrange neutral members[6], as given in Table I (for a comment on the particle names used, see the text following the introduction of Table III).

| Table I: Antisymmetric Octet ($\mathbf{8_2}$) | | | | | | | |
|---|---|---|---|---|---|---|---|
| $J^{PC}$ | $S$ | $I$ | $I_3$ | $Q$ | Name | wave functions |
| $1^{--}$ | 0 | 1 | +1 | +1 | $\sigma^+_-$ | $\sqrt{\frac{1}{3}}\,(\pi^+\,\pi^0 - \pi^0\,\pi^+) - \sqrt{\frac{1}{6}}\,\left(\bar{K}^0\,K^+ - K^+\,\bar{K}^0\right)$ |
| | | | 0 | 0 | $\sigma^0_-$ | $\sqrt{\frac{1}{3}}\,(\pi^+\,\pi^- - \pi^-\,\pi^+) - \sqrt{\frac{1}{12}}\left(\bar{K}^0\,K^0 - K^0\,\bar{K}^0\right)$ $-\sqrt{\frac{1}{12}}\,(K^-\,K^+ - K^+\,K^-)$ |
| | | | −1 | −1 | $\sigma^-_-$ | $\sqrt{\frac{1}{3}}\,(\pi^0\,\pi^- - \pi^-\,\pi^0) - \sqrt{\frac{1}{6}}\,(K^-\,K^0 - K^0\,K^-)$ |
| $1^{--}$ | 0 | 0 | 0 | 0 | $\lambda^0_-$ | $\frac{1}{2}\left(\bar{K}^0\,K^0 - K^0\,\bar{K}^0\right) - \frac{1}{2}\,(K^-\,K^+ - K^+\,K^-)$ |

It is clear that they come with an eigenvalue of $C = -1$, and so they cannot couple to $\pi\eta$. Another way to explain this is to note that, for an antisymmetric $\pi\eta$ to belong to an octet, the wave function for its isosinglet partner must also be antisymmetric, but such a wave function can only come with $C = -1$ as shown in Table I. An antisymmetric $\pi\eta$ can appear in the $\mathbf{10} \oplus \overline{\mathbf{10}}$ representation, because it is devoid of an isosinglet member.

A summary of the various $J^{PC}$'s allowed for all the representations in $8 \otimes 8$ is given in Table II.

We conclude that the absence of the $\eta\pi$ decay mode of the $\pi_1(1600)$ is due to its being a member of an octet of flavor $SU(3)$. This conclusion holds, whatever its constituent quark content may be, hybrid or four-quark system.

Now we allow for deviations from ideal mixing in the pseudoscalar meson nonet. Then, the $\pi_1(1400)$ has a small but finite chance to decay into $\eta'\pi$ and the $\pi_1(1600)$ to decay into $\eta\pi$. The mixing is, however, not large enough to reverse the experimental observation of a large coupling of the $\pi_1(1400)$ to $\eta\pi$ and of the $\pi_1(1600)$ to $\eta'\pi$.

| Table II: $SU(3)$ Multiplets and their Composition | | |
|---|---|---|
| $SU(3)$ Multiplet | $J^{PC}$ or $J^P$ | Composition |
| Singlet ($\mathbf{1}$) | even$^{++}$ | $q\bar{q}$, $q\bar{q}+g$, $q\bar{q}+q\bar{q}$ |
| Symmetric Octet ($\mathbf{8_1}$) | even$^{++}$ | $q\bar{q}$, $q\bar{q}+g$, $q\bar{q}+q\bar{q}$ |
| Antisymmetric Octet ($\mathbf{8_2}$) | odd$^{--}$ | $q\bar{q}$, $q\bar{q}+g$, $q\bar{q}+q\bar{q}$ |
| multiplet $\mathbf{20}$ ($\mathbf{10}\oplus\overline{\mathbf{10}}$) | odd$^{-}$ | $q\bar{q}+q\bar{q}$ (14 strange states) |
| | odd$^{-+}$ | $q\bar{q}+q\bar{q}$ (3 non-strange states) |
| | odd$^{--}$ | $q\bar{q}+q\bar{q}$ (3 non-strange states) |
| Multiplet 27 | even$^{++}$ | $q\bar{q}+q\bar{q}$ |

A system of $\eta\pi$ in a $p$-wave has been the subject of long-standing investigation by theorists for a number of years. The first reference we are aware of, on this topic, is that of B.-W. Lee *et al.* [7] who pointed out in 1964 that such a system must necessarily belong to an 'icosuplet' (i.e. $\mathbf{10}\oplus\overline{\mathbf{10}}$). H. J. Lipkin [8] and F. E. Close and H. J. Lipkin [9] pointed out the exotic nature of a $\phi\pi$ and a $p$-wave $\eta\pi$ state in the same vein. For further comments on the $p$-wave $\eta\pi$ system, the reader is referred to the later articles by different authors [10].

But why does the $\pi_1(1400)$ decay into $\eta\pi$? As shown above, the requirement of Bose symmetry, coupled with standard $SU(3)$ isoscalar factors, leads to the conclusion that the $p$-wave $\eta\pi$ final state is not contained in a flavor octet. We have to use a higher SU(3) representation.

The product of two $SU(3)$ octets breaks up into[11]

$$\mathbf{8}\otimes\mathbf{8} \;=\; \mathbf{27} \;\oplus\; \mathbf{10} \;\oplus\; \overline{\mathbf{10}} \;\oplus\; \mathbf{8_1} \;\oplus\; \mathbf{8_2} \;\oplus\; \mathbf{1} \qquad (2)$$

The 27-plet has even spin only, hence we have to use the $\mathbf{10}\oplus\overline{\mathbf{10}}$ representation. The appropriate normalized wave functions can be written down using the $SU(3)$ isoscalar factors[11]; the results are tabulated in Table III. One sees that all the wave functions are antisymmetric under the interchange of two states. In order that the Bose symmetrization holds for the entire wave function, one must demand that the spatial symmetry - orbital angular momentum

- be odd. We see that a $p$-wave $\pi\eta$ system must belong to the $\mathbf{10} \oplus \overline{\mathbf{10}}$ representation.

One notes that the particle names in Table III are derived from their counterparts in the Baryon sector but with lower-case letters. Thus a $I^G(J^{PC}) = 1^-(1^{-+})$ $\pi_1(1400)$ is given the name $\sigma_+(1400)$ whereby emphasis is placed on the four-quark nature of the object. A $\rho$ is a generic name for a state with $I^G(J^{PC}) = 1^+(1^{--})$ but the name $\sigma_-$ signifies its four-quark character. The counterparts to $K^*$ and $\omega$ are likewise denoted $\xi$ and $\lambda$, respectively.

Table IIIa: multiplet $\mathbf{20}$ ($\mathbf{10} \oplus \overline{\mathbf{10}}$)—strange members

| $J^P$ | $S$ | $I$ | $I_3$ | $Q$ | Name | wave functions |
|---|---|---|---|---|---|---|
| $1^-$ | +2 | 0 | 0 | +1 | $\bar{\omega}^+$ | $-\sqrt{\frac{1}{2}}\,(K^+ K^0 - K^0 K^+)$ |
| $1^-$ | +1 | 3/2 | +3/2 | +2 | $\delta^{++}$ | $\frac{1}{\sqrt{2}}\,(\pi^+ K^+ - K^+ \pi^+)$ |
| | | | +1/2 | +1 | $\delta^+$ | $\sqrt{\frac{1}{6}}\,(\pi^+ K^0 - K^0 \pi^+) + \sqrt{\frac{1}{3}}\,(\pi^0 K^+ - K^+ \pi^0)$ |
| | | | -1/2 | 0 | $\delta^0$ | $\sqrt{\frac{1}{3}}\,(\pi^0 K^0 - K^0 \pi^0) + \sqrt{\frac{1}{6}}\,(\pi^- K^+ - K^+ \pi^-)$ |
| | | | -3/2 | $-1$ | $\delta^-$ | $\frac{1}{\sqrt{2}}\,(\pi^- K^0 - K^0 \pi^-)$ |
| $1^-$ | +1 | 1/2 | +1/2 | +1 | $\bar{\xi}^+$ | $-\sqrt{\frac{1}{6}}\,(\pi^+ K^0 - K^0 \pi^+) + \sqrt{\frac{1}{12}}\,(\pi^0 K^+ - K^+ \pi^0)$ $+\frac{1}{2}\,(K^+ \eta - \eta K^+)$ |
| | | | -1/2 | 0 | $\bar{\xi}^0$ | $-\sqrt{\frac{1}{12}}\,(\pi^0 K^0 - K^0 \pi^0) + \sqrt{\frac{1}{6}}\,(\pi^- K^+ - K^+ \pi^-)$ $+\frac{1}{2}\,(K^0 \eta - \eta K^0)$ |
| $1^-$ | $-1$ | 1/2 | +1/2 | 0 | $\xi^0$ | $\sqrt{\frac{1}{6}}\,(\pi^+ K^- - K^- \pi^+) - \sqrt{\frac{1}{12}}\left(\pi^0 \bar{K}^0 - \bar{K}^0 \pi^0\right)$ $+\frac{1}{2}\left(\bar{K}^0 \eta - \eta \bar{K}^0\right)$ |
| | | | -1/2 | $-1$ | $\xi^-$ | $\sqrt{\frac{1}{12}}\,(\pi^0 K^- - K^- \pi^0) - \sqrt{\frac{1}{6}}\left(\pi^- \bar{K}^0 - \bar{K}^0 \pi^-\right)$ $+\frac{1}{2}\,(K^- \eta - \eta K^-)$ |
| $1^-$ | $-1$ | 3/2 | +3/2 | +1 | $\bar{\delta}^+$ | $\frac{1}{\sqrt{2}}\left(\pi^+ \bar{K}^0 - \bar{K}^0 \pi^+\right)$ |
| | | | +1/2 | 0 | $\bar{\delta}^0$ | $\sqrt{\frac{1}{6}}\,(\pi^+ K^- - K^- \pi^+) + \sqrt{\frac{1}{3}}\left(\pi^0 \bar{K}^0 - \bar{K}^0 \pi^0\right)$ |
| | | | -1/2 | $-1$ | $\bar{\delta}^-$ | $\sqrt{\frac{1}{3}}\,(\pi^0 K^- - K^- \pi^0) + \sqrt{\frac{1}{6}}\left(\pi^- \bar{K}^0 - \bar{K}^0 \pi^-\right)$ |
| | | | -3/2 | $-2$ | $\bar{\delta}^{--}$ | $\frac{1}{\sqrt{2}}\,(\pi^- K^- - K^- \pi^-)$ |
| $1^-$ | $-2$ | 0 | 0 | $-1$ | $\omega^-$ | $\sqrt{\frac{1}{2}}\left(\bar{K}^0 K^- - K^- \bar{K}^0\right)$ |

| Table IIIb: multiplet **20** ($\mathbf{10} \oplus \overline{\mathbf{10}}$)—non-strange members | | | | | | |
|---|---|---|---|---|---|---|
| $J^{PC}$ | $S$ | $I$ | $I_3$ | $Q$ | Name | wave functions |
| $1^{-+}$ | 0 | 1 | +1 | +1 | $\sigma_+^+$ | $\frac{1}{\sqrt{2}}\,(\pi^+\,\eta - \eta\,\pi^+)$ |
| | | | 0 | 0 | $\sigma_+^0$ | $\frac{1}{\sqrt{2}}\,(\pi^0\eta - \eta\pi^0)$ |
| | | | -1 | $-1$ | $\sigma_+^-$ | $\frac{1}{\sqrt{2}}\,(\pi^-\,\eta - \eta\,\pi^-)$ |
| $1^{--}$ | 0 | 1 | +1 | +1 | $\sigma_-^+$ | $\sqrt{\frac{1}{6}}\,(\pi^+\,\pi^0 - \pi^0\,\pi^+) + \sqrt{\frac{1}{3}}\,\left(\bar{K}^0\,K^+ - K^+\,\bar{K}^0\right)$ |
| | | | 0 | 0 | $\sigma_-^0$ | $\sqrt{\frac{1}{6}}\,(\pi^+\,\pi^- - \pi^-\,\pi^+) + \sqrt{\frac{1}{6}}\,\left(\bar{K}^0\,K^0 - K^0\,\bar{K}^0\right)$ $+\sqrt{\frac{1}{6}}\,(K^-\,K^+ - K^+\,K^-)$ |
| | | | -1 | -1 | $\sigma_-^-$ | $\sqrt{\frac{1}{6}}\,(\pi^0\,\pi^- - \pi^-\,\pi^0) + \sqrt{\frac{1}{3}}\,(K^-\,K^0 - K^0\,K^-)$ |

It can be shown that antisymmetric wave functions (under the interchange of the two particles) occur for the $\mathbf{8_2}$, $\mathbf{10}$ and $\overline{\mathbf{10}}$ representations, all others being symmetric. It is obvious that a $p$-wave $\pi\eta$ system belongs to the $\mathbf{10} \oplus \overline{\mathbf{10}}$ representation, as shown in Table III.

Let us now discuss the consequences of a assignement of the $\pi_1(1400)$ to the $\mathbf{10} \oplus \overline{\mathbf{10}}$ multiplet.

- The multiplet **20** does not contain an isosinglet. This has to be contrasted to the octet $\pi(1600)$ for which we expect an isosinglet partner $\eta_1(1600)$. The $\eta_1(1600)$ could be searched for in four-pion final states like $a_1(1260) + \pi$ via S-wave or $\pi(1300) + \pi$ via P-wave.

- There are no non-strange mesons with charge 2. A possible $Q = \pm 2$ candidate has been reported in the $\pi\pi$ channel[12] which, if confirmed, would constitute the first example of the multiplet **27**.

- The multiplet **20** contains mesons with strangeness 2. They could be searched for in reactions like

$$K^+\,p \to K^+\,K_S^0\,\Sigma^+ \tag{3}$$

The negative strangeness of the $\Sigma^+$ identifies the $K_S^0$ as $K^0$. The state with strangeness $S = -2$ is much more difficult to find since the $K^- K_S^0$ has a $S = 0$ and a $S = 2$ component.

- The multiplet **20** contains strange mesons with isospin 3/2. The maximal third component has $I_3 = 3/2$ and mesons with 2 units of charge are supposed to exist. The search for such particles again requires K$^+$ beams:

$$K^+ \, p \rightarrow K^+ \, \pi^+ \, \Lambda \tag{4}$$

We conclude that—in the limit in which the $\eta$ is a pure $SU(3)$ octet and the $\eta'$ is a pure singlet, and assuming that flavor $SU(3)$ is conserved in the decay—the exotic $\pi_1(1400)$ state with quantum numbers $I^G(J^{PC}) = 1^-(1^{-+})$ and seen to decay into $\eta\pi$ and not into $\eta'\pi$ must be a four-quark state belonging to the multiplet **20**, a $\mathbf{10} \oplus \overline{\mathbf{10}}$ representation of flavor $SU(3)$, because of the requirement of Bose symmetrization on a $p$-wave $\pi\eta$ system. The $\pi_1(1600)$ is seen to decay primarily into $\eta'\pi$ and not into $\eta\pi$. This state must belong to an $SU(3)$ octet and could thus be an exotic meson with a valence gluon in it, i.e. $(q\bar{q} + g)$, although it could just as well be a four-quark state. A verification of new meson configuratons, in particular the verification of mesons in $SU(3)$ decuplets requires an intense positively charged Kaon beam. Such experiments could be carried out at the RF-separated $K^\pm$ beam currently at IHEP/Protvino, the 50-GeV Japan Hadron Facility at KEK or at the GSI-SIS200 project.

Achasov and Shestakov[13] has recently worked out the nature of exotic resonances with $I^G(J^{PC}) = 1^-(1^{-+})$ in the process $\rho\pi \rightarrow \eta\pi$. They derive a set of unitarized and analytic amplitudes for the process in which the $s$-channel loop diagrams contain the intermediate states $\rho\pi$, $\eta\pi$, $\eta'\pi$ and $K^*\bar{K} + \bar{K}^*K$. They consider both $\pi_1(1400)$ and $\pi_1(1600)$ in their model and find that an exotic state coupling to $\eta\pi$ must belong to the $\mathbf{10} \oplus \overline{\mathbf{10}}$ reprsentation of flavor $SU(3)$. However, their model requires a decay mode $\pi_1(1400) \rightarrow \rho\pi$. We note that—experimentally—the $\pi_1(1400)$ is seen only in the $\eta\pi$ decay channel. If the $\rho$ exchange is absent in the reaction $\pi^- p \rightarrow \pi_1^0(1400)n$, then it can proceed only via $b_1(1235)$- and $\rho_2(1700)$-Reggeon[14] exchanges. So the intensity of the $\pi_1^0(1400)$ production falls off as $1/s$ in comparison to that of the $a_2^0(1320)$ production. Let $R$ be the ratio of the cross section for $\pi^- p \rightarrow \pi_1^0(1400)n$ to that for $\pi^- p \rightarrow a_2^0(1320)n$. Then the suggested model predicts

$$R(\text{KEK}) : R(\text{BNL}) : R(\text{IHEP}) : R(\text{CERN}) = 2.7 \, : \, 1.0 \, : 0.49 \, : \, 0.18 \tag{5}$$

Here the relevant center-of-mass energy squared ($s$) corresponds to the $\pi^- p$ interactions at 6.3 GeV/$c$ (KEK[15]), 18 GeV/$c$ (BNL[1]), 37 GeV/$c$ (IHEP/Protvino[3]) and 100 GeV/$c$ (CERN[16]). Unfortunately, not all of the requisite cross sections are known experimentally.

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